PHYSICS

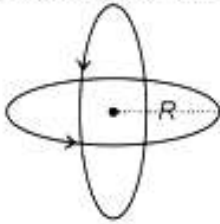### SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.
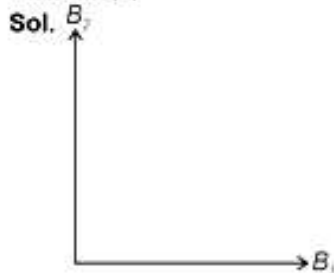
Choose the correct answer:

1. Two rings of equal radius R arranged perpendicular to each other with common centre at C, carrying equal current I. Find magnetic field at C.

(1) $\dfrac{\mu_0 I}{2R}$

(2) $\dfrac{\mu_0 I}{R}$

(3) $\sqrt{2}\,\dfrac{\mu_0 I}{R}$

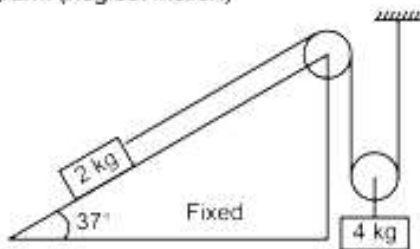(4) $\dfrac{\mu_0 I}{\sqrt{2}R}$

Answer (4)

Sol.

$B_1 = \dfrac{\mu_0 I}{2R}\,\hat{i},\ B_2 = \dfrac{\mu_0 I}{2R}\,\hat{j}$

$B_C = \dfrac{\mu_0 I}{\sqrt{2}R}$

2. Find the acceleration of 2 kg block shown in the diagram. (neglect friction)

(1) $\dfrac{4g}{15}$

(2) $\dfrac{2g}{15}$

(3) $\dfrac{g}{15}$

(4) $\dfrac{2g}{3}$

Answer (1)

Sol. For 2 kg block

$T - 2g\sin 37 = 2a$ ...(i)

For 4 kg block

$4g - 2T = \dfrac{4a}{2}$

$2g - T = a$ ...(ii)

$T = (2g - a)$

$2g - a - 2g \times \dfrac{3}{5} = 2a$

$3a = 2g \times \dfrac{2}{5}$

$\boxed{a = \dfrac{4g}{15}}$

3. A particle of mass m is projected with speed v at an angle of $30°$ with the horizontal, find its angular momentum about point of projection when it reaches its maximum height.

(1) $\dfrac{mv^3}{16g}$

(2) $\sqrt{3}\,\dfrac{mv^3}{16g}$

(3) $\dfrac{mv^3}{3g}$

(4) $\sqrt{3}\,\dfrac{mv^3}{8g}$

Answer (2)

Sol. Velocity at maximum height = $v\cos 30°$

$\therefore\ L = m(v\cos 30)\,H$

$= mv\left(\dfrac{\sqrt{3}}{2}\right) \times \dfrac{v^2 \sin^2 30}{2g}$
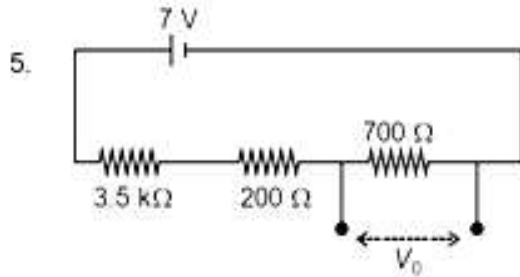
$= \sqrt{3}\,\dfrac{mv^3}{16\,g}$

4. The ratio of kinetic energy & potential energy in 5th excited state of Hydrogen atom is

(1) -2

(2) 2

(3) $-\dfrac{1}{2}$

(4) $\dfrac{1}{2}$

Answer (3)

Sol. Kinetic energy : Potential energy = 1 : –2

5.

In given circuit find potential difference across 700 Ω resistance (i.e. V_0).
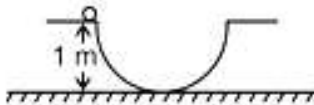
(1) 2 V   (2) 0.5 V

(3) 1.1 V   (4) Zero

Answer (3)

Sol. $i = \dfrac{7}{3.5\,k + 0.9\,k\Omega} = \dfrac{7}{4.4\,k}$

$V_0 = i \times 700\ \Omega = \dfrac{7}{4.4\,k} \times .7\,k = \dfrac{4.9}{4.4} = 1.1$ V

6. A ball is released from a height of 1 m on a smooth hemispherical surface as shown. Find its velocity when it is at a height of 0.5 m. (Take $g = 10$ m/s^2)

(1) 20 m/s   (2) 10 m/s
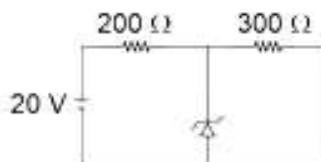
(3) $\sqrt{10}$ m/s   (4) 5 m/s

Answer (3)

Sol. By conservation of mechanical energy

$mg(1) = \dfrac{1}{2}mv^2 + mg(0.5)$

$v^2 = 10$

$v = \sqrt{10}$ m/s

7. Find current through zener diode if its breakdown voltage is 5 V.

(1) 58.33 mA   (2) 25 mA

(3) 28.33 mA   (4) 20.23 mA

Answer (1)

Sol. $i_{battery} = \dfrac{(20-5)}{200} = \dfrac{15}{200}$ A

$i_{300\Omega} = \dfrac{5}{300}$ A

$\therefore i_{zener} = \dfrac{15}{200} - \dfrac{5}{300}$

$= 58.33$ mA

8. Ball released from height 10 m strikes ground and rebounds height 5 m. Find impulse imparted by ground while collision, given mass of ball is 100 g. (Take $g = 10$ m/s^2)

(1) $(\sqrt{2}-1)$ Ns   (2) $(\sqrt{2}+2)$ Ns

(3) $(2\sqrt{2}-1)$ Ns   (4) $(\sqrt{2}+1)$ Ns

Answer (4)

Sol. $v_1 = \sqrt{2g10}$

$v_2 = \sqrt{2g5}$

$\vec{I} = \Delta\vec{p}$

$I = 0.1\{\sqrt{2g10} + \sqrt{2g5}\}$

$= 0.1\{10\sqrt{2} + 10\}$

$= (\sqrt{2}+1)$ Ns

9. Potential due to electric dipole on axial position at distance r from dipole is proportional to (assume $r \gg$ length of dipole)

(1) $\dfrac{1}{r}$ (2) $\dfrac{1}{r^3}$

(3) $\dfrac{1}{r^2}$ (4) r

Answer (3)

Sol.
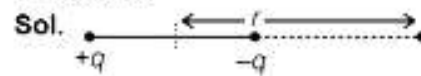
$|E| = \dfrac{2kP}{r^3}$

$E = -\dfrac{dv}{dr}, \ v \propto \dfrac{1}{r^2}$

10. Maximum wavelength of light source such that photoelectron can be ejected from material of work function 3 eV is

(1) 2133.3 A   (2) 3133.3 A

(3) 4133.3 A   (4) 313.3 A

Answer (3)

Sol. $\lambda = \dfrac{12400}{3} = 4133.3$ A

11. A long wire carrying current $\sqrt{2}A$ is placed in uniform magnetic field of 3×10^{-5} T. If magnetic field is perpendicular to wire, find the magnetic force on unit length of wire.

   (1) 3×10^{-4} N

   (2) $3\sqrt{2} \times 10^{-5}$ N

   (3) 3×10^3 N

   (4) Zero

Answer (2)

Sol.

$F = i\,L\,B\,\sin\theta$

$= \sqrt{2} \times 1 \times 3 \times 10^{-5} \times \sin 90$

$F = 3\sqrt{2} \times 10^{-5}$ N

12. If the area of cross-section is halved and length of wire having young's modulus Y is doubled, then its young's modulus will become

   (1) Y

   (2) $4\,Y$

   (3) $\dfrac{Y}{2}$

   (4) $\dfrac{Y}{4}$

Answer (1)

Sol. Young's modulus is property of material of wire and it is independent of geometrical factors.

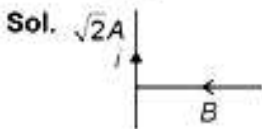13. In an electric transformer, 220 V is applied on primary coil having number of turn 100. Find output current through 3 Ω resistance if number of secondary turn is 10.
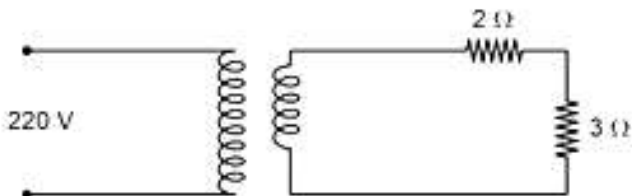
   (1) 4 A

   (2) 4.4 A

   (3) 2 A

   (4) 2.2 A

Answer (2)

Sol. $\dfrac{V_i}{V_0} = \dfrac{N_i}{N_0} \Rightarrow \dfrac{220}{V_0} = \dfrac{100}{10}$

$V_0 = 22$ V

$\therefore\ I_0 = \dfrac{22}{5} = 4.4$ A

14. Find the temperature of H_2 gas at which its *rms* speed is equal to that of O_2 at 47°C.

   (1) 20°C

   (2) −20°C

   (3) −253°C

   (4) 17°C

Answer (3)

Sol. $V_{rms} = \sqrt{\dfrac{3RT}{M}}$

$\dfrac{T}{2} = \dfrac{320}{32}$

$T = 20$ K

$\therefore\ T = -253°C$

15. In AC circuit with source voltage $\varepsilon = 20 \sin 1000\,t$ is connected to series L − R circuit whose power factor is $\dfrac{1}{\sqrt{2}}$. If $E = 25 \sin 2000\,t$, the new power factor is

   (1) $\dfrac{2}{\sqrt{5}}$

   (2) $\dfrac{1}{\sqrt{5}}$

   (3) $\dfrac{1}{\sqrt{3}}$

   (4) $\sqrt{\dfrac{3}{5}}$

Answer (2)

Sol.

Old
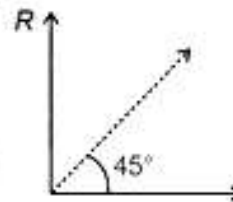
$L\omega = 1000\,L \qquad\qquad \Rightarrow\ L\omega = R.$

New

$R = 1000\,L$
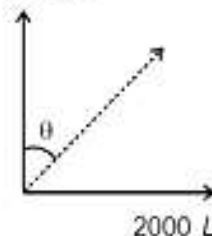
$\cos\theta = \dfrac{R}{Z}$

$= \dfrac{1000\,L}{\sqrt{(1000\,L)^2 + (2000\,L)^2}}$

$= \dfrac{1}{\sqrt{1+4}} = \dfrac{1}{\sqrt{5}}$

16. In an electromagnetic wave the electric field is given as $\vec{E} = E_0 \sin(\omega t - kz)\hat{i}$ the corresponding magnetic field will be

(1) $E_0 C \sin(\omega t - kz)\hat{j}$

(2) $\dfrac{E_0}{C} \sin(\omega t - kz)\hat{j}$

(3) $\dfrac{E_0}{C} \cos(\omega t - kz)\hat{i}$

(4) $\dfrac{E_0}{C} \sin(\omega t - kz)\hat{i}$

Answer (2)

Sol. $\vec{E} \times \vec{B}$ is along +z axis

$B_0 = \dfrac{E_0}{C}$

$\therefore \quad B = \dfrac{E_0}{C} \sin(\omega t - kz)\hat{j}$

17. At a point away from planet of radius 6400 km, the gravitational potential and field are -6.4×10^7 SI units and 6.4 SI units respectively. Find height of that point above surface of planet.

(1) 3000 km

(2) 6400 km

(3) 3600 km

(4) 9400 km

Answer (3)

Sol. $\dfrac{GM}{r} = 6.4 \times 10^7 \qquad ...(i)$

$\dfrac{GM}{r^2} = 6.4 \qquad\qquad ...(ii)$

$r = \dfrac{6.4 \times 10^7}{6.4}$

$= 10^7$ m

$= 10{,}000$ km

$R + h = 10{,}000$

$h = 10{,}000 - 6400 = 3600$ km

18. A wire has resistance of 60 Ω at temperature 27°C. When it is connected to a 220 V dc supply, a current 2.75 A flows through it at a certain temperature. Find the value of temperature, if coefficient of thermal resistance (α) is 2×10^{-4}/°C.

(1) 1694°C

(2) 1500°C

(3) 1000°C

(4) 1200°C

Answer (1)

Sol. Final resistance $(R) = \dfrac{V}{I} = 80$ Ω

then, $R = R_0(1 + \alpha \Delta T)$

$80 = 60\left(1 + 2 \times 10^{-4} \Delta T\right)$

$\Delta T = 1666.67$

$T - 27$

$T = 1693.66$

$= 1694$°C

19. Match the two columns.

	Column 1		Column 2
P.	Surface tension	1.	$[ML^2T^{-2}]$
Q.	Viscosity	2.	$[ML^2T^{-1}]$
R.	Angular momentum	3.	$[ML^{-1}T^{-1}]$
S.	Rotational kinetic energy	4.	$[ML^0T^{-2}]$

(1) P–1, Q–2, R–3, S–4

(2) P–4, Q–3, R–2, S–1

(3) P–1, Q–3, R–4, S–2

(4) P–4, Q–2, R–1, S–3

Answer (2)

Sol. $S = \dfrac{F}{L}$

$\Rightarrow [S] = [MT^{-2}]$

$F = nA\dfrac{dv}{dx}$

$\Rightarrow \eta = \dfrac{MLT^{-2} \cdot T}{L^2} = ML^{-1}T^{-1}$

$\dot{L} = \dot{r} \times \dot{p}$

$\Rightarrow L = [ML^2T^{-1}]$

$KE = \dfrac{1}{2}I\omega^2$

$\Rightarrow KE = ML^2T^{-2}$

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. A block of mass 2 kg is placed on a disc which is rotating at constant angular velocity 4 rad/s. Find the friction force (in N) between block and disc if block is not sliding.
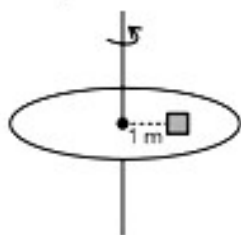
Answer (32)

Sol. Block is not slipping, so

$f = mr\omega^2 = 2 \times 1 \times 16 = 32$

22. Distance between virtual image, which is of twice of size of object placed in front of mirror and object is 45 cm. Magnitude of focal length of mirror is _____ cm.

Answer (30)
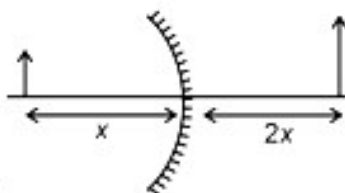
Sol.

$|m| = \left|\dfrac{v}{u}\right| = 2$

$|v| = |2u|$

$n + 2n = 45$

$n = 15$ cm

$u = -15$

$v = 30$

$\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$\dfrac{1}{30} + \dfrac{1}{-15} = \dfrac{1}{f}$

$\dfrac{1-2}{30} = \dfrac{-1}{30} = \dfrac{1}{f}$

$\Rightarrow f = 30$ cm

23. A particle is having uniform acceleration. If its displacement from t to $(t + 1)$ second is 120 m and change in velocity is 50 m/s. Find its displacement (in m) in $(t + 2)$ second.

Answer (170)

Sol. $\Delta v = a(t + 1 - t)$

$\therefore \quad a = 50$ m/s^2

$s = u + \dfrac{a}{2}(2(t + 1) - 1)$

$120 = u + \dfrac{50}{2}(2t + 1)$

$\therefore \quad u = 120 - 25(2t + 1)$

In $(t + 2)^{th}$ second

$s' = u + \dfrac{a}{2}(2(t + 2) - 1)$

$= u + 25(2t + 3)$

$= 120 - 25(2t) - 25 + 25(2t) + 75$

$s' = 170$ m

24. A uniform disc of mass 5 kg and radius 2 m is rotating with 10 rad/s. Now another identical disc is gently placed on first disc. Because of friction, both disc acquire common angular velocity. Loss of kinetic energy in process is _____ J.

Answer (250)

Sol. COAM gives $I\omega_0 = 2I\omega$

$\omega = \dfrac{\omega_0}{2}$

Loss in KE $= \dfrac{1}{2}I\omega_0^2 - \dfrac{1}{2}(2I)\left(\dfrac{\omega_0}{2}\right)^2$

$= \dfrac{1}{4}I\omega_0^2$

$= \dfrac{1}{4} \times 5 \times \dfrac{2}{2} \times 100 = 250$ J

25. Two cell one of emf 8 V, internal resistance 2 Ω and other of emf 2 V and internal resistance 4 Ω are connected as shown in figure.

Find potential difference (in V) across point AC.
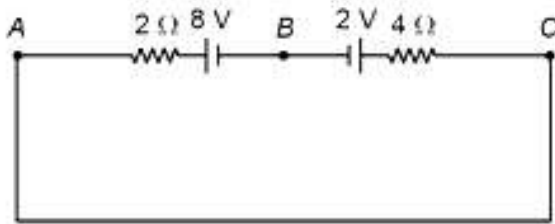
Answer (0)

Sol. Current in circuit $(I) = \dfrac{8-2}{6} = 1$ A.

So, $V_C - 4(1) - 2 + 8 - 2(1) = V_A$

$V_C - 6 - 2 + 8 = V_A$

$V_C - V_A = 0$ V

26. Electron in an hydrogen atom is excited to an energy level having energy –0.85 eV. Find the number of possible transitions it can make while de-excitation.

Answer (6)

Sol. $-0.85 = \dfrac{-13.6}{n^2}$

$n = 4$

$\therefore$ Number of transitions $= \dfrac{4 \times 3}{2} = 6$

27. Energy stored in circuit 1 is E. If capacitors in circuit 1 and circuit 2 are connected in parallel as shown, the energy stored becomes $\dfrac{xE}{6}$, find x.

Circuit 1

Circuit 2
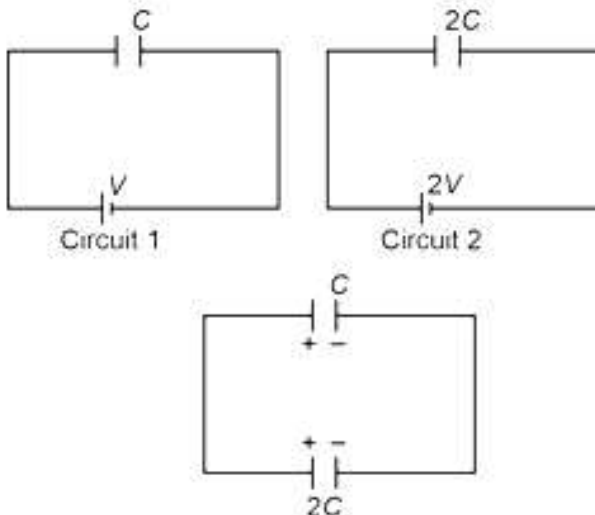
Answer (50)

Sol. Charge on $C_1 = CV$

Charge on $C_2 = 4CV$

When connected in parallel

$V_c = \dfrac{5V}{3}$

$\therefore Q_1' = \dfrac{5}{3}CV, \quad Q_2' = \dfrac{10}{3}CV$

$\because E = \dfrac{1}{2}CV^2$

$E' = \dfrac{25}{18}CV^2 + \dfrac{25}{9}CV^2$

$\dfrac{25}{6}CV^2 = \dfrac{50E}{6}$

$\therefore x = 50$

28.
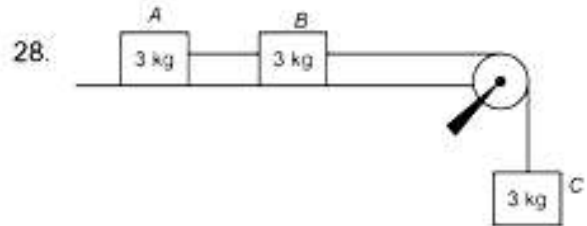
If wire BC has Young's modulus of $Y = 2 \times 10^{11}$ N/m^2 and cross section area of 5×10^{-4} cm^2. Find strain in wire BC (in unit of 10^{-4})

Answer (20)

Sol. $a = \dfrac{3}{9}g$, For C, $3g - T = 3a = (3)\dfrac{3}{9}g$

$T = 2g = 20$ N

$\dfrac{\sigma}{\epsilon} = Y$

$\dfrac{\sigma}{Y} = \epsilon \Rightarrow \dfrac{20}{5 \times 10^{-8} \times 2 \times 10^{11}}$

$= 2 \times 10^{-3}$

$= 20 \times 10^{-4} \Rightarrow 20$

29.

30.

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. What is the name of given reaction?
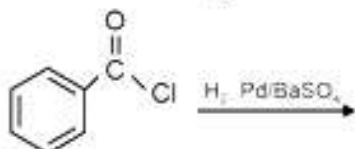
   (1) Etard reaction

   (2) Stephen's reaction

   (3) Wolff Kishner reduction

   (4) Rosenmund reaction

Answer (4)

Sol. Acyl chloride is hydrogenated over catalyst, palladium or barium sulphate. This reaction is called Rosenmund reaction.
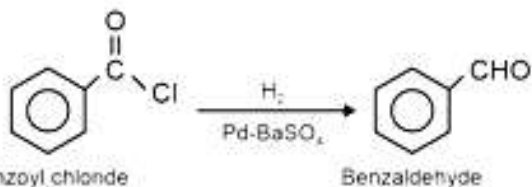
2. Which of the given compound will not give Fehling test?

   (1) Lactose

   (2) Maltose

   (3) Sucrose

   (4) Glucose

Answer (3)

Sol. Sucrose is non-reducing sugar. It does not reduce Fehling solution.

3. Find final product of reaction given below

(i) Mg, dry ether
(ii) CO_2, H^+
(iii) NH_3, Δ
(iv) Br_2, KOH
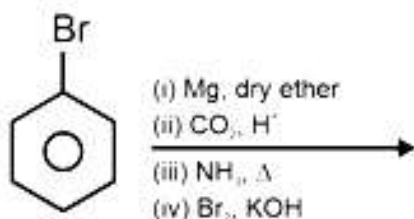
Answer (1)

Sol.
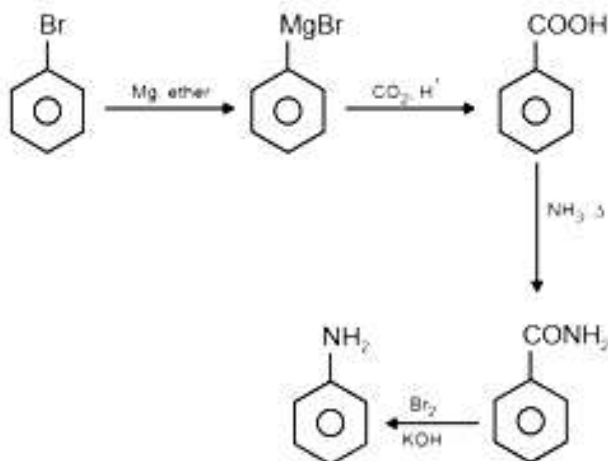
4. Which of the following has allylic halogen?

   (1) CH = CH — CH_2 — CH_2 — Cl

   (2)

   (3)

   (4) CH_3 — CH — CH_3 with Cl attached to middle carbon

Answer (3)

Sol. The carbon next to an alkene is known as allylic carbon and halogen attached to allylic carbon is known as allylic halogen.

e.g.

allylic carbons

allylic carbons
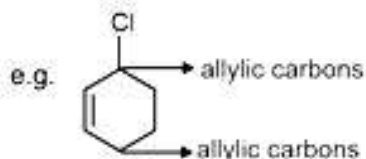
5. Which of the following compound or ion is most stable?

(1)   (2)

(3)   (4)

Answer (3)

Sol. is most stable due to aromatic character. It has $2\pi e^-$ and follow $(4n + 2)\pi e^-$ Huckel rule.

6. Which of the following set contains both diamagnetic ions?

(1) Ni^{2+}; Cu^{2+} (2) Eu^{3+}; Gd^{3+}

(3) Cu^+; Zn^{2+} (4) Ce^{4+}; Pm^{3+}

Answer (3)

Sol. Cu : $4s^1 3d^{10}$; Cu^+ : $4s^0 3d^{10}$

Zn : $4s^2 3d^{10}$; Zn^{2+} : $4s^0 3d^{10}$

7. Consider the following sequence of reactions
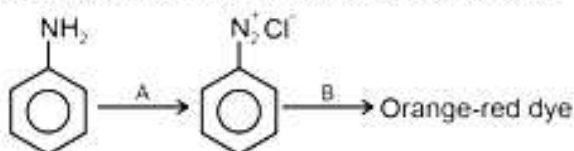
Select the option with correct A and B respectively.

(1) HNO_3, Phenol   (2) $NaNO_2/HCl$, Phenol

(3) HNO_3, Aniline   (4) $NaNO_2/HCl$, Aniline

Answer (2)

Sol.

Orange-red solid
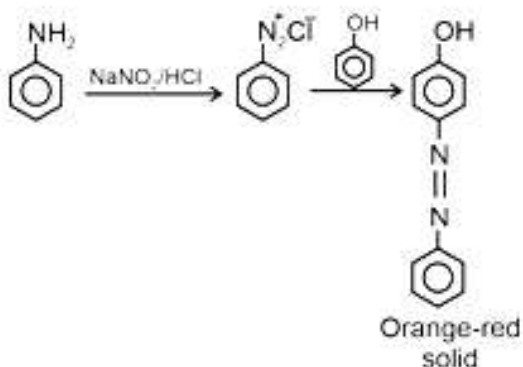
8. Which of the following is the correct structure for the given IUPAC name "3-Methylpent-2-enal"

(1)   (2)

(3)   (4)
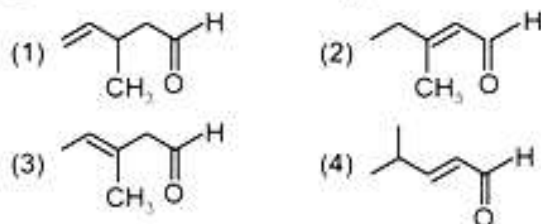
Answer (2)

Sol.
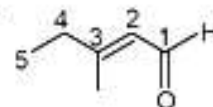
3-Methylpent-2-enal

Functional group should get lowest possible number.

9. The group number of Unununium is

(1) 11   (2) 12

(3) 6   (4) 14

Answer (1)

Sol. Group number = 11 (Atomic number = 111)

10. What is the Geometry of Aluminium chloride in aqueous solution?

(1) Square planar   (2) Octahedral

(3) Tetrahedral   (4) Square pyramidal

Answer (2)

Sol. $AlCl_3$ exists as

$[Al(H_2O)_6]Cl_3$ in aqueous solution.

11. Statement-I: For hydrogen atom, 3p and 3d are degenerate.

Statement-II: Degenerate orbitals have same energy.

(1) Both statement-I and II are correct

(2) Both statement-I and II are incorrect

(3) Statement-I is correct, statement-II is incorrect

(4) Statement-I is incorrect, statement-II is correct

Answer (1)

Sol. For hydrogen atom energy of orbitals only depends on value of principal quantum number

$1s < 2s = 2p < 3s = 3p = 3d < 4s = 4p = 4d = 4f$

Degenerate orbitals have same energy.

12. Consider the following sequence of reactions

$$CH_3 - C \equiv CH \xrightarrow{Na} A \xrightarrow{B} CH_3 - C \equiv C - CH_2 - CH_2 - CH_3$$

Select A and B respectively

(1) $CH_3 - CH = CH_2$, $CH_3 - CH_2 - Cl$

(2) $CH_3 - C \equiv CNa$, $CH_3 - CH_2 - Cl$

(3) $CH_3 - C \equiv CNa$, $CH_3 - CH_2 - CH_2 - Cl$

(4) $CH_3 - C \equiv C - CH_3$, $CH_3 - CH_2 - CH_2 - Cl$

Answer (3)

Sol. $CH_3 - C \equiv CH \xrightarrow{Na} \underset{(A)}{CH_3 - C \equiv CNa} + H_2(g)$

$$\downarrow CH_3 - CH_2 - CH_2 - Cl \ (B)$$

$$CH_3 - C \equiv C - CH_2 - CH_2 - CH_3$$
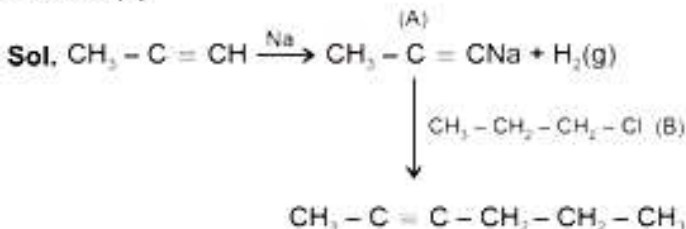
13. Choose the correct option.

Column-I (Molecule)	Column-II (Shape)
a. BrF_5	(i) See-saw
b. H_2O	(ii) T-shape
c. ClF_3	(iii) Bent
d. SF_4	(iv) Square pyramidal

(1) a(iv), b(iii), c(ii), d(i) (2) a(iv), b(iii), c(i), d(ii)

(3) a(iii), b(iv), c(ii), d(i) (4) a(iii), b(iv), c(i), d(ii)

Answer (1)

Sol. BrF_5 – Square pyramidal

H_2O – Bent

ClF_3 – T-shape

SF_4 – See-saw

14. Assertion (A) : While moving from N to P covalent radius increases significantly but from As to Bi only a small increase is observed.

Reason (R) : For a particular oxidation state covalent radii and ionic generally radii increases down the group.

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(3) (A) is correct but (R) is incorrect

(4) (A) is incorrect but (R) is correct

Answer (2)

Sol. Covalent and ionic (in a particular state) radii increases in size down the group. There is a considerable increase in covalent radius from N to P. However, from As to Bi only a small increase in covalent radii is observed. This is due to the presence of completely filled d and f-orbitals in heavier elements. (lanthanoid contraction)

15. Match the following and select the correct option.

List I	List II
a. Mn^{2+}	(i) $3d^3\ 4s^1$
b. V^+	(ii) $3d^5\ 4s^0$
c. Cr^+	(iii) $3d^5\ 4s^0$
d. Fe^{2+}	(iv) $3d^4\ 4s^1$

(1) a → (i), b → (ii), c → (iii), d → (iv)

(2) a → (iv), b → (iii), c → (ii), d → (i)

(3) a → (ii), b → (i), c → (ii), d → (iii)

(4) a → (ii), b → (i), c → (iii), d → (iv)

Answer (3)

Sol. Mn^{2+} : $3d^5\ 4s^0$

V^+ : $3d^3\ 4s^1$

Cr^+ : $3d^5\ 4s^0$

Fe^{2+} : $3d^6\ 4s^0$

16. What happen to freezing point of benzene, when small amount of naphthalene is added to benzene?

(1) Increases

(2) Decreases

(3) Remains unchanged

(4) First decreases and then increases

Answer (2)

Sol. When small amount of naphthalene is added to benzene, depression in freezing point takes place and freezing point of benzene decreases.

17. A mixture is heated with dilute H_2SO_4 and the lead acetate paper turns black by the evolved gas. The mixture contains

(1) Sulphite (2) Sulphide

(3) Sulphate (4) Thiosulphate

Answer (2)
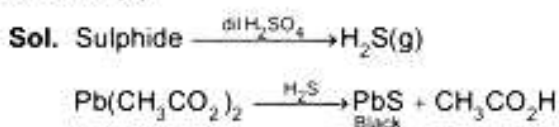
Sol. Sulphide $\xrightarrow{dil\,H_2SO_4} H_2S(g)$

$$Pb(CH_3CO_2)_2 \xrightarrow{H_2S} \underset{Black}{PbS} + CH_3CO_2H$$

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Find out sum of coefficients of all the species involved in balance equation

$$2MnO_4^- + I^- \xrightarrow{Alkaline\ Medium}$$

Answer (9)

Sol. $I^- + H_2O + 2MnO_4^- \rightarrow 2MnO_2 + 2OH^- + IO_3^-$
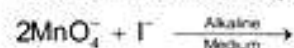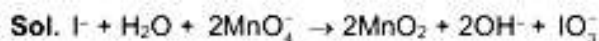
Sum of coefficients = 9

22. Find work done in following cyclic process (in J).
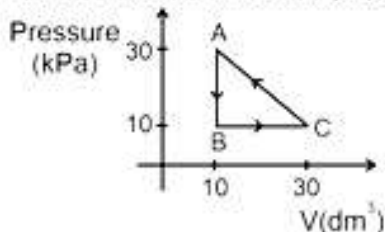
Answer (200)

Sol. Work done in cyclic process

= area inside the figure

$= \dfrac{1}{2} \times 20 \times 20 = 200$ J

23. Maximum number of hybrid orbitals formed when $2s$ and $2p$ orbitals of a single atom are mixed.

Answer (4)

Sol. When $2s$ and $2p$ orbitals are mixed, maximum 4 hybrid orbitals are formed
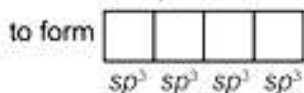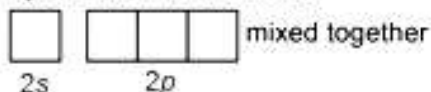
4 sp^3 hybrid orbitals are formed of same energy.

24. The rate of first order reaction is 0.04 mol lit^{-1} sec^{-1} at 10 sec and 0.03 mol lit^{-1} sec^{-1} at 20 sec. Calculate half-life of first order reaction (in sec).

Answer (24)

Sol. $\dfrac{0.04}{0.03} = \dfrac{k \times C_0 e^{-k(10)}}{k \times C_0 e^{-k(20)}} = e^{10k}$

$10k = \ln\left(\dfrac{4}{3}\right)$

$k = \dfrac{1}{10} \ln\left(\dfrac{4}{3}\right)$

$t_{\frac{1}{2}} = \dfrac{\ln 2}{k}$

$= \dfrac{\ln 2}{\ln\left(\dfrac{4}{3}\right)} \times 10$

$= 24$ sec.

25. The number of atoms in a silver plate having area 0.05 cm^2 and thickness 0.05 cm is _____ × 10^{19}. Density of silver is 7.9 g/cm^3

Answer (11)

Sol. Volume = Area × Thickness

$= 0.05 \times 0.05$ cm^3

$= 0.0025$ cm^3

Mass of silver = 7.9 × 0.0025 g

Moles of silver $= \dfrac{7.9 \times 0.0025}{108}$

Number of silver atoms

$= \dfrac{7.9 \times 0.0025}{108} \times 6.022 \times 10^{23}$

$\Rightarrow$ Number of silver atoms $= 0.001101 \times 10^{23}$

$= 11.01 \times 10^{19}$

26. The ratio of magnitude of potential energy and kinetic energy for 5th excited state of hydrogen atom is

Answer (2)

Sol. According to Bohr model, PE = –2KE

27. 250 mL solution of CH_3COONa of molarity 0.35 M is prepared. What is the mass of CH_3COONa required in grams? (Nearest integer)

Answer (7)

Sol. Molarity $= \dfrac{\text{Number of moles of solute}}{\text{Volume of solution in litre}}$

Moles of solute $= \dfrac{\text{Weight}}{\text{Molecular weight}}$

$0.35 = \dfrac{W}{MW(CH_3COONa)} \times \dfrac{1000}{25}$

$W = \dfrac{0.35 \times 82 \times 250}{1000}$

$W = \dfrac{7175}{1000} = 7.175$ g

Mass of CH_3COONa required to prepare 250 mL of 0.35 M solution is 7.175 g.

28. The K_{sp} of $Mg(OH)_2$ is 1×10^{-12}, 0.01 M Mg^{2+} ion will precipitate at the limiting pH equal to _____ (at 25°C).

Answer (9)

Sol. $Mg(OH)_2(s) \rightleftharpoons Mg^{2+}(aq) + 2OH^-(aq)$

$K_{sp} = [Mg^{2+}][OH^-]^2$

$K_{sp} = 0.01 [OH^-]^2$

$\dfrac{1 \times 10^{-12}}{0.01} = [OH^-]^2$

$[OH^-] = \sqrt{1 \times 10^{-10}}$

$[OH^-] = 10^{-5}$

pOH = 5

pH = 14 –5 = 9

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. In an arithmetic progression if sum of 20 terms is 790 and sum of 10 terms is 145, then $S_{15} - S_5$ is (when S_n denotes sum of n terms)

   (1) 400   (2) 395

   (3) 385   (4) 405

Answer (2)

Sol. $S_{20} = \dfrac{20}{2}[2a + 19d] = 790$

$2a + 19d = 79$ ...(1)

$S_{10} = \dfrac{10}{2}[2a + 9d] = 145$

$2a + 9d = 29$ ...(2)

from (1) and (2) $a = -8$, $d = 5$

$S_{15} - S_5 = \dfrac{15}{2}[2a + 14d] - \dfrac{5}{2}[2a + 4d]$

$= \dfrac{15}{2}[-16 + 70] - \dfrac{5}{2}[-16 + 20]$

$= 405 - 10$

$= 395$

2. If the foot of perpendicular from (1, 2, 3) to the line $\dfrac{x+1}{2} = \dfrac{y-2}{5} = \dfrac{z-4}{1}$ is (α, β, γ) then find $\alpha + \beta + \gamma$

   (1) 6   (2) 5.8

   (3) 4.8   (4) 5

Answer (2)

Sol.

$(\alpha - 1) \times 2 + (\beta - 2) \times 5 + (\gamma - 3) \times 1 = 0$

$2\alpha + 5\beta + \gamma - 15 = 0$

Also, P lie on line

$\Rightarrow \alpha + 1 = 2\lambda$

$\beta - 2 = 5\lambda$

$\gamma - 4 = \lambda$

$\Rightarrow 2(2\lambda - 1) + 5(5\lambda + 2) + \lambda + 4 - 15 = 0$

$\Rightarrow 4\lambda + 25\lambda + \lambda - 2 + 10 + 4 - 15 = 0$

$30\lambda - 3 = 0$

$\Rightarrow \lambda = \dfrac{1}{10}$

$\Rightarrow \alpha + \beta + \gamma = (2\lambda - 1) + (5\lambda + 2) + (\lambda + 4)$

$= 8\lambda + 5 = \dfrac{8}{10} + 5 = 5.8$

3. $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \dfrac{n^3}{(n^2 + k^2)(n^2 + 3k^2)}$

   (1) $\dfrac{\pi}{2\sqrt{3}} - \dfrac{\pi}{8}$ (2) $\dfrac{\pi}{2\sqrt{3}} + \dfrac{\pi}{8}$

   (3) $\dfrac{\pi}{2} - \dfrac{\pi}{\sqrt{3}}$ (4) $\dfrac{\pi}{\sqrt{3}} - \dfrac{\pi}{4}$

Answer (1)

Sol. $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \dfrac{n^3}{n^4\left(1 + \dfrac{k^2}{n^2}\right)\left(1 + \dfrac{3k^2}{n^2}\right)}$

$= \displaystyle\lim_{n\to\infty} \dfrac{1}{n} \sum_{k=1}^{n} \dfrac{1}{\left(1 + \dfrac{k^2}{n^2}\right)\left(1 + \dfrac{3k^2}{n^2}\right)}$

$= \displaystyle\int_{0}^{1} \dfrac{dx}{3(1 + x^2)\left(\dfrac{1}{3} + x^2\right)}$

$= \displaystyle\int_{0}^{1} \dfrac{1}{3} \times \dfrac{3}{2} \dfrac{(x^2 + 1) - \left(x^2 + \dfrac{1}{3}\right)}{(1 + x^2)\left(x^2 + \dfrac{1}{3}\right)} dx$

$= \dfrac{1}{2} \displaystyle\int_{0}^{1} \left[\dfrac{1}{x^2 + \left(\dfrac{1}{\sqrt{3}}\right)^2} - \dfrac{1}{1 + x^2}\right] dx$

$= \dfrac{1}{2}\left[\sqrt{3}\tan^{-1}(\sqrt{3}x)\right]_{0}^{1} - \dfrac{1}{2}\left(\tan^{-1}x\right)_{0}^{1}$

$= \dfrac{\sqrt{3}}{2}\left(\dfrac{\pi}{3}\right) - \dfrac{1}{2}\left(\dfrac{\pi}{4}\right) = \dfrac{\pi}{2\sqrt{3}} - \dfrac{\pi}{8}$

4. The value of maximum area possible of a $\triangle ABC$ such that $A(0, 0)$ and $B(x, y)$ and $C(-x, y)$ such that $y = -2x^2 + 54x$ is (in sq. unit)

(1) 5800　　　　　　　(2) 5832

(3) 5942　　　　　　　(4) 6008

Answer (2)

Sol.

Area of $\triangle$

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x & y & 1 \\ -x & y & 1 \end{vmatrix}$$

$$\Rightarrow \left|\frac{1}{2}(xy + xy)\right| = |xy|$$

Area $(\triangle) = |xy| = \left|x(-2x^2 + 54x)\right|$

$$\frac{d(\triangle)}{dx} = \left|(-6x^2 + 108x)\right| \Rightarrow \frac{d\triangle}{dx} = 0 \text{ at } x = 0 \text{ and } 18$$

$\Rightarrow$ at $x = 0$, minima

and at $x = 18$ maxima

Area $(\triangle) = \left|18(-2(18)^2 + 54 \times 18)\right| = 5832$

5. The range of r for which circles $(x + 1)^2 + (y + 2)^2 = r^2$ and $x^2 + y^2 - 4x - 4y + 4 = 0$ coincide at two distinct points

(1) $3 < r < 7$　　　　(2) $5 < r < 9$

(3) $\frac{1}{2} < r < 4$　　　(4) $0 < r < 3$

Answer (1)

Sol. If two circles intersect at two distinct points

$\Rightarrow |r_1 - r_2| < C_1C_2 < r_1 + r_2$

$|r - 2| < \sqrt{9 + 16} < r + 2$

$|r - 2| < 5$　　　and $r + 2 > 5$

$-5 < r - 2 < 5$　　　$r > 3$　　　...(2)

$-3 < r < 7$　　　　　　　　...(1)

From (1) and (2)

$3 < r < 7$

6. An ellipse whose length of minor axis is equal to half of length between foci, then eccentricity is

(1) $\frac{7}{2}$　　　　　　　(2) $\sqrt{17}$

(3) $\frac{2}{\sqrt{5}}$　　　　　　　(4) $\frac{3}{\sqrt{7}}$

Answer (3)

Sol. $\because ae = 2b$

$\therefore \frac{4b^2}{a^2} = e^2$

Or $4(1 - e^2) = e^2$

$\therefore 4 = 5e^2 \Rightarrow e = \frac{2}{\sqrt{5}}$

7. If $g'\left(\frac{3}{2}\right) = g'\left(\frac{1}{2}\right)$ and

$f(x) = \frac{1}{2}[g(x) + g(2 - x)]$ and $f'\left(\frac{3}{2}\right) = f'\left(\frac{1}{2}\right)$ then

(1) $f''(x) = 0$ has exactly one root in $(0, 1)$

(2) $f''(x) = 0$ has no root in $(0, 1)$

(3) $f''(x) = 0$ has at least two roots in $(0, 2)$

(4) $f''(x) = 0$ has 3 roots in $(0, 2)$

Answer (3)

Sol.

$f'(x) = \frac{g'(x) - g'(2 - x)}{2}, f'\left(\frac{3}{2}\right) = \frac{g'\left(\frac{3}{2}\right) - g'\left(\frac{1}{2}\right)}{2} = 0$

Also $f'\left(\frac{1}{2}\right) = \frac{g'\left(\frac{1}{2}\right) - g'\left(\frac{3}{2}\right)}{2} = 0, f'(1) = 0$

$\Rightarrow f'\left(\frac{3}{2}\right) = f'\left(\frac{1}{2}\right) = 0$

$\Rightarrow$ roots in $\left(\frac{1}{2}, 1\right]$ and $\left(1, \frac{3}{2}\right)$

$\Rightarrow f''(x)$ is zero at least twice in $\left(\frac{1}{2}, \frac{3}{2}\right)$

8. The domain of $y = \cos^{-1}\left|\frac{2 - |x|}{4}\right| + (\log(3 - x))^{-1}$ is $[-\alpha, \beta) - \{\gamma\}$, then value of $\alpha + \beta + \gamma = ?$

(1) 9　　　　　　　(2) 12

(3) 11　　　　　　　(4) 10

Answer (3)

Sol. $-1 \le \left| \dfrac{2-|x|}{4} \right| \le 1$

$\Rightarrow \left| \dfrac{2-|x|}{4} \right| \le 1$

$\Rightarrow -1 \le \dfrac{2-|x|}{4} \le 1$

$-4 \le 2-|x| \le 4$

$-6 \le -|x| \le 2$

$-2 \le |x| \le 6$

$|x| \le 6$

$\Rightarrow x \in [-6, 6]$ ...(1)

Now, $3 - x \ne 1$

And $x \ne 2$ ...(2)

and $3 - x > 0$

$x < 3$ ...(3)

From (1), (2) and (3)

$\Rightarrow x \in [-6, 3] - \{2\}$

$\alpha = 6$

$\beta = 3$

$\gamma = 2$

$\alpha + \beta + \gamma = 11$

9. If $y = f(x)$ is solution of differential equation $(x^2 - 1)\, dy = \left((x^3 + 1) + \sqrt{1 - x^2} \right) dx$ and $y(0) = 2$ then find $y\left(\dfrac{1}{2} \right)$.

(1) $\dfrac{13}{7} - \dfrac{\pi}{2} + \ln 5$ (2) $\dfrac{15}{7} + \dfrac{\pi}{3} + \ln 2$

(3) $\dfrac{17}{8} + \dfrac{\pi}{6} - \ln 2$ (4) $\dfrac{18}{7} - \dfrac{\pi}{6} + \ln 3$

Answer (3)

Sol. $\dfrac{dy}{dx} = \dfrac{(x+1)(x^2 - x + 1) + \sqrt{(1-x)(1+x)}}{(x-1)(x+1)}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{x(x-1)+1}{(x-1)} + \sqrt{\dfrac{(1-x)(1+x)}{(x-1)^2(x+1)^2}}$

$\dfrac{dy}{dx} = x + \dfrac{1}{x-1} + \dfrac{1}{\sqrt{(1-x)(1+x)}}$

$\Rightarrow dy = x\, dx + \dfrac{1}{(x-1)} dx + \dfrac{dx}{\sqrt{1-x^2}}$

$\Rightarrow y = \dfrac{x^2}{2} + \ln|x-1| + \sin^{-1} x + c$

at $x = 0, y = 2 \Rightarrow 2 = c$

$\Rightarrow y = \dfrac{x^2}{2} + \ln|x-1| + \sin^{-1} x + 2$

$y\left(\dfrac{1}{2} \right) = \dfrac{17}{8} + \dfrac{\pi}{6} - \ln 2$

10. Given $x^2 - 70x + \lambda = 0$ with positive roots α and β where one of the root is less than 10 and $\dfrac{\lambda}{2}$ and $\dfrac{\lambda}{3}$ are not integers then find value of $\dfrac{\sqrt{\alpha - 1} + \sqrt{\beta - 1}}{|\alpha - \beta|}$ is equal to

(1) $\dfrac{1}{5}$ (2) $\dfrac{1}{12}$

(3) $\dfrac{1}{60}$ (4) $\dfrac{1}{70}$

Answer (1)

Sol. Given : $x^2 - 70x + \lambda = 0$

$\Rightarrow$ Let roots be α and β

$\Rightarrow \beta = 70 - \alpha$

$\lambda = \alpha(70 - \alpha)$

λ is not divisible by 2 and 3

$\Rightarrow \alpha = 5, \beta = 65$

$\Rightarrow \dfrac{\sqrt{5-1} + \sqrt{65-1}}{|60|} = \left| \dfrac{4+8}{60} \right| = \dfrac{1}{5}$

11. A line passes through $(9, 0)$, making angle $30°$ with positive direction of x-axis. It is rotated by angle of $15°$ with respect to $(9, 0)$. Then one of the equation of new line is

(1) $y = (2 + \sqrt{3})(x - 9)$ (2) $y = (2 - \sqrt{3})(x - 9)$

(3) $y = 2(x - 9)$ (4) $y = -(x - 9)$

Answer (2)

Sol.

$Eq^n : y - 0 = \tan 15°\,(x-9) \Rightarrow y = (2-\sqrt{3})(x-9)$

$Eq^n : y - 0 = \tan 45°\,(x-9) \Rightarrow y = (x-9)$

Option (B) is correct

12. For a non-zero complex number z satisfying $z^2 + i\bar{z} = 0$, then value of $|z|^2$ is

(1) 1 (2) 2

(3) 3 (4) 4

Answer (1)

Sol. $z^2 = -i\bar{z}$

$|z^2| = |-i\bar{z}|$

$|z^2| = |z|$

$|z|^2 - |z| = 0$

$|z|(|z|-1) = 0$

$|z| = 0$ (not acceptable)

$\therefore |z| = 1$

$\therefore |z|^2 = 1$

13. If $|\vec{a}| = 1$, $|\vec{b}| = 4$

$\vec{a}\cdot\vec{b} = 2$ and $\vec{c} = 2(\vec{a}\times\vec{b}) - 3\vec{b}$

Then the angle between $\vec{b}$ and $\vec{c}$ is

(1) $\theta = \cos^{-1}\left(\dfrac{-\sqrt{3}}{2}\right)$ (2) $\theta = \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

(3) $\theta = \cos^{-1}\left(\dfrac{1}{2}\right)$ (4) $\theta = \cos^{-1}\left(\dfrac{-1}{2}\right)$

Answer (1)

Sol. Given $|\vec{a}| = 1$, $|\vec{b}| = 4$, $\vec{a}\cdot\vec{b} = 2$

$\vec{c} = 2(\vec{a}\times\vec{b}) - 3\vec{b}$

Dot product with $\vec{a}$ on both sides

$\vec{c}\cdot\vec{a} = -6$ ... (1)

Dot product with $\vec{b}$ on both sides

$\vec{b}\cdot\vec{c} = -48$ ... (2)

$\vec{c}\cdot\vec{c} = 4|\vec{a}\times\vec{b}|^2 + 9|\vec{b}|^2$

$|\vec{c}|^2 = 4\left[|\vec{a}|^2\cdot|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2\right] + 9|\vec{b}|^2$

$|\vec{c}|^2 = 4\left[(1)(4)^2 - (4)\right] + 9(16)$

$|\vec{c}|^2 = 4[12] + 144$

$|\vec{c}|^2 = 48 + 144$

$|\vec{c}|^2 = 192$

$\therefore \cos\theta = \dfrac{\vec{b}\cdot\vec{c}}{|\vec{b}||\vec{c}|}$

$\cos\theta = \dfrac{-48}{\sqrt{192}\cdot 4}$

$\cos\theta = \dfrac{-48}{8\sqrt{3}\cdot 4}$

$\cos\theta = \dfrac{-3}{2\sqrt{3}}$

$\cos\theta = \dfrac{-\sqrt{3}}{2}$ $\Rightarrow$ $\theta = \cos^{-1}\left(\dfrac{-\sqrt{3}}{2}\right)$

14. Given set $S = \{0, 1, 2, 3, ..., 10\}$. If a random ordered pair (x, y) of elements of S is chosen, then find probability that $|x - y| > 5$

(1) $\dfrac{30}{121}$ (2) $\dfrac{31}{121}$

(3) $\dfrac{62}{121}$ (4) $\dfrac{64}{121}$

Answer (1)

Sol. If $x = 0$, $y = 6, 7, 8, 9, 10$

If $x = 1$, $y = 7, 8, 9, 10$

If $x = 2$, $y = 8, 9, 10$

If $x = 3$, $y = 9, 10$

If $x = 4$, $y = 10$

If $x = 5$, $y = $ no possible value

Total possible ways = $(5 + 4 + 3 + 2 + 1) \times 2$

= 30

Required probability = $\dfrac{30}{11\times 11} = \dfrac{30}{121}$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Number of integral terms in the binomial expansion of $(7^{1/2} + 11^{1/6})^{824}$ is

Answer (138)

Sol. $T_{n+1} = {}^nC_r 11^{\frac{r}{6}} \cdot 7^{\frac{824-r}{2}}$

For integral term

6 should divide r

and $\dfrac{824-r}{2}$ must be integer

$\Rightarrow$ 2 most divide r

$\Rightarrow$ r divisible by 6

$\Rightarrow$ possible values of $r \in \{0, 1, 2, \dots 824\}$

$\Rightarrow$ For integer terms

$r \in \{0, 6, 12, \dots 822\}$ $(822 = 0 + (n-1)6 \Rightarrow n = 138)$

= 138 terms

22. $9\int\limits_0^9 \left[\sqrt{\dfrac{10x}{x+1}}\right] dx$ is equal to (where [ ] represents greatest integer function)

Answer (155)

Sol. $I = 9\int\limits_0^9 \left[\sqrt{\dfrac{10x}{x+1}}\right] dx$

$= 9\left[\int\limits_0^{1/9} 0\, dx + \int\limits_{1/9}^{2/3} dx + \int\limits_{2/3}^{9} 2dx\right]$

$= 9\left[\dfrac{2}{3} - \dfrac{1}{9} + 2\left(9 - \dfrac{2}{3}\right)\right]$

$= 9\left[\dfrac{5}{9} + 2 \times \dfrac{25}{3}\right]$

$= 5 + 6 \times 25$

$= 5 + 150$

$= 155$

23. In a class there are 40 students. 16 passed in Chemistry, 20 passed in Physics, 25 passed in Mathematics. 15 students passed in both Mathematics and Physics. 15 students passed in both Mathematics and Chemistry and 10 students passed in both Physics and Chemistry. Find the maximum number of students that passed in all the subjects.

Answer (19)

Sol. $n(C) = 16$, $n(P) = 20$, $n(M) = 25$

$n(M\cap P) = n(M\cap C) = 15$, $n(P\cap C) = 10$,

$n(M\cap C\cap P) = x$.
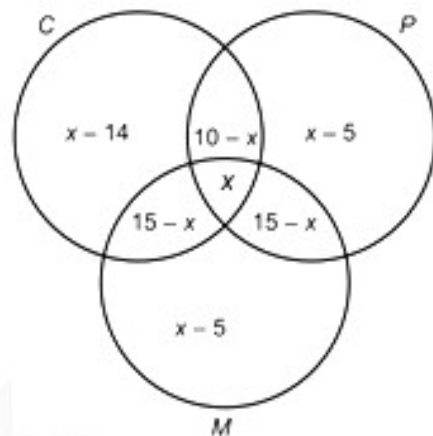
$n(C\cup P\cup M) \leq n(U) = 40$

$n(C\cup P\cup M) = n(C) + n(P) + n(M) - n(C\cup M) - n(P\cup M) - n(C\cap P) + n(C\cap P\cap M)$

$40 \geq 16 + 20 + 25 - 15 - 15 - 10 + x$

$40 \geq 61 - 40 + x$

$19 \geq x$

So maximum number of students that passed all the exams is 19.

24. For the following data table

x_i	f_i
0 – 4	2
4 – 8	4
8 – 12	7
12 – 16	8
16 – 20	6

Find the value of 20 M (where M is median of the data)

Answer (245)

Sol.

x_i	f_i	c.f.
0 – 4	2	2
4 – 8	4	6
8 – 12	7	13
12 – 16	8	21
16 – 20	6	27

$N = \sum f = 27$

$\left(\dfrac{N}{2}\right) = \dfrac{27}{2} = 13.5$

So, we have median lies in the class 12 – 16

$l_1 = 12, f = 8, h = 4, c.f. = 13$

So, here we apply formula

$M = l_1 + \dfrac{\dfrac{N}{2} - c.f.}{f} \times h = 12 + \dfrac{13.5 - 13}{8} \times 4$

$= 12 + \dfrac{.5}{2}$

$M = \dfrac{24.5}{2} = 12.25$
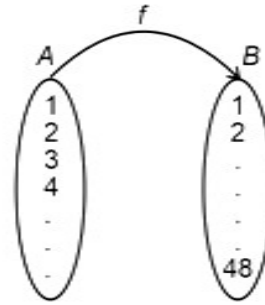
$20\, M = 20 \times 12.25$

$= 245$

25. Set $A = \{1, 2, 3, 4, 5, 6, 7\}$

If number of functions from set A to power set of A can be expressed as m^n (m is least integer). Find $m + n$.

Answer (51)

Sol. $n\, P(A) = 2^7 = 128$

$f : A \to B$

Number of function $= 128 \times 128 ....128 = 128^7$

$= \left(2^7\right)^7 = 2^{49}$

$\Rightarrow\ m^n = 2^{49}$

$\therefore\ m + n = 49 + 2 = 51$

26.

27.

28.

29.

30.

❏ ❏ ❏