

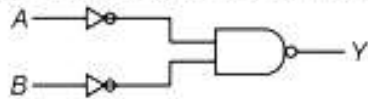
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The logical circuit shown below is equivalent to

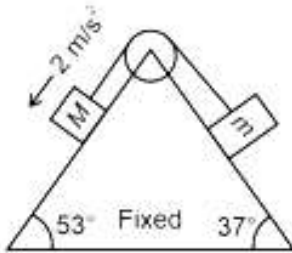


- (1) NAND (2) NOR
(3) AND (4) OR

Answer (4)

Sol. Output, $Y = \overline{\overline{A} \cdot \overline{B}} = A + B$

2. The block M of mass 10 kg is having acceleration 2 m/s^2 in the direction shown. Find mass (m) of the other block.



- (1) 2.5 kg (2) 7.5 kg
(3) 12.5 kg (4) 5 kg

Answer (3)

Sol. $a = \frac{(M \sin 53 - m \sin 37)g}{(M + m)}$

$$m = \frac{15}{2} \text{ kg}$$

3. If the percentage error in measuring length and diameter of a wire is 0.1% each, then the percentage error of the resistance of the wire is

- (1) 0.3% (2) 0.2%
(3) 0.1% (4) 0.4%

Answer (1)

Sol. $R = \frac{\rho l}{A}$

$$R = \frac{4\rho l}{\pi d^2}$$

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} + 2 \frac{\Delta d}{d}$$

$$= 0.3 \%$$

4. 4 identical particles of mass m each are placed at 4 corners of a square. The gravitational force exerted on one of the mass by other masses is

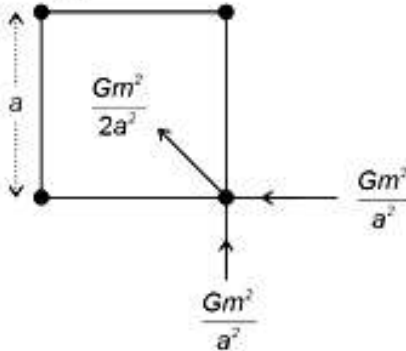
$$\left[\frac{2\sqrt{2} + 1}{32} \right] \frac{Gm^2}{l^2}$$

The distance of side of square is :

- (1) $2l$ (2) $4l$
(3) $\frac{l}{2}$ (4) l

Answer (2)

Sol.



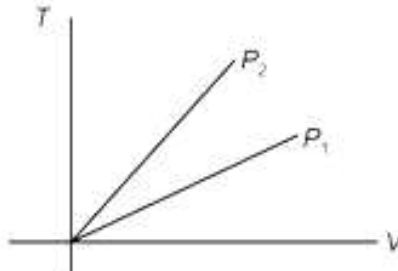
$$\Rightarrow F_{\text{net}} = \frac{Gm^2}{a^2} \left[\frac{1}{2} + \sqrt{2} \right]$$

$$= \frac{2\sqrt{2} + 1}{2} \frac{Gm^2}{a^2}$$

$$\Rightarrow 2a^2 = 32l^2$$

$$\Rightarrow a = 4l$$

5. $T - V$ graph is given for two different pressures P_1 & P_2 . Then



- (1) $P_2 > P_1$ (2) $P_1 = P_2$
(3) $P_2 < P_1$ (4) $P_2 \leq P_1$

Answer (1)

Sol. $T - V$ graph: Straight line

\Rightarrow Isobaric

Also, slope $\propto P$

$\Rightarrow P_2 > P_1$

6. For a 1 - D motion, relation between position x and time t is $t = \alpha x^2 + \beta x$. Find the relation between velocity v and acceleration a .

(1) $a = \alpha v$ (2) $a = -2\alpha v$

(3) $a = -2\alpha v^3$ (4) $a = 2\alpha v^2$

Answer (3)

Sol. $t = \alpha x^2 + \beta x$

$$\Rightarrow \frac{dt}{dx} = 2\alpha x + \beta = \frac{1}{v}$$

$$\Rightarrow v = \frac{1}{2\alpha x + \beta}$$

$$\text{Also, } a = \frac{v dv}{dx} = \frac{-1}{2\alpha x + \beta} \left[\frac{1}{2\alpha x + \beta} \right]^2 \cdot 2\alpha$$

$$\Rightarrow a = \frac{-2\alpha}{(2\alpha x + \beta)^3}$$

$$= -2\alpha v^3$$

$$\Rightarrow \boxed{a = -2\alpha v^3}$$

7. Two resistances having coefficient of variation of resistivity α_1 and α_2 are having equal resistance. Equivalent temperature coefficient of resistivity in series and parallel combination are.

(1) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$ (2) $\alpha_1 + \alpha_2, \alpha_1 + \alpha_2$

(3) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$ (4) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$

Answer (4)

Sol. $R_1 = R_0(1 + \alpha_1 T)$

$R_2 = R_0(1 + \alpha_2 T)$

$$\alpha_s = \frac{R_0(1 + \alpha_1 T) + R_0(1 + \alpha_2 T) - 2R_0}{2R_0 T}$$

$$\alpha_s = \frac{\alpha_1 + \alpha_2}{2}$$

$$\alpha_{||} = \frac{\frac{R_0(1 + \alpha_1 T) + R_0(1 + \alpha_2 T)}{2} - R_0}{\frac{R_0 T}{2}}$$

$$\alpha_{||} = \frac{(\alpha_1 + \alpha_2) T}{2} \cdot \frac{2}{T} = \frac{\alpha_1 + \alpha_2}{2}$$

8. An artillery of mass M_1 , fires a shell of mass M_2 . At the time of firing the ratio of kinetic energy is

(1) $\frac{M_2}{M_1}$

(2) $\frac{M_1 + M_2}{M_1}$

(3) $\frac{M_1 + M_2}{M_2}$

(4) $\frac{M_1}{M_1 + M_2}$

Answer (1)

Sol. By conservation of momentum

$$P_1 = -P_2$$

$$\frac{K_1}{K_2} = \frac{P_1^2}{2M_1} \times \frac{2M_2}{P_2^2} = \frac{M_2}{M_1}$$

9. The fundamental frequency of closed organ pipe is equal to the frequency of first overtone of open organ pipe of length 60 cm. The length of closed organ pipe is

(1) 45 cm

(2) 30 cm

(3) 15 cm

(4) 60 cm

Answer (3)

$$\text{Sol. } \frac{V}{4L_1} = 2 \left(\frac{V}{2L_2} \right)$$

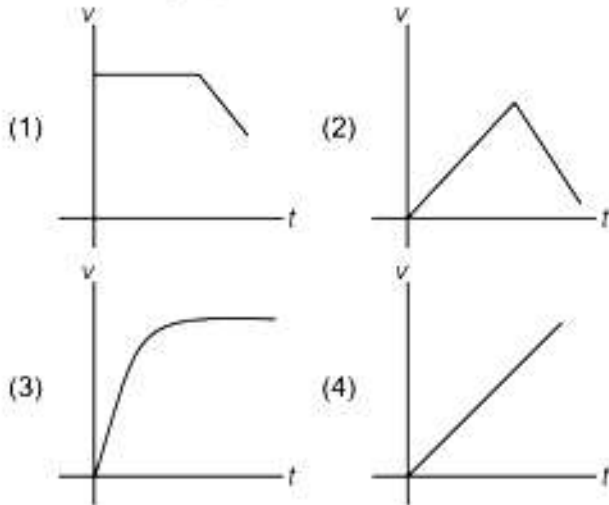
L_1 = Length of closed organ pipe

L_2 = Length of open organ pipe

$$L_2 = 4L_1$$

$$L_1 = \frac{L_2}{4} = 15 \text{ cm}$$

10. When a small spherical ball is dropped into a long cylindrical pipe filled with glycerine, then what will be the v versus t graph?



Answer (3)

Sol. $F = mg - B - 6\pi\eta r v$

Where B : Buoyancy

$$\Rightarrow m \frac{dv}{dt} = \underbrace{(mg - B)}_{\text{constant}} - \underbrace{(6\pi\eta r)}_{\text{constant}} v$$

$$\Rightarrow v = v_0 [1 - e^{-ct}], \quad c : \text{constant}$$

11. Force F depends on distance (x) and time(t) as

$$F = ax^2 + bt^{\frac{1}{2}}, \text{ find dimension of } \frac{b^2}{a}$$

- (1) $M'L^2T^{-3}$ (2) $M'L^{-3}T^3$
(3) $M'L^3T^{-3}$ (4) $M'L^2T$

Answer (3)

Sol. From dimensional analysis

$$[a] = \frac{[F]}{[L^2]} = \frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$$

$$[b] = \frac{[F]}{\left[\frac{1}{T^{\frac{1}{2}}}\right]} = \frac{MLT^{-2}}{T^{\frac{1}{2}}} = MLT^{-\frac{5}{2}}$$

Then dimension of $\frac{b^2}{a} = \frac{M^2L^2T^{-5}}{ML^{-1}T^{-2}}$

$$\left[\frac{b^2}{a}\right] = [ML^3T^{-3}]$$

12. Two charges q & $3q$ are placed at a distance r from each other. Find the distance from q where electric field is zero.

- (1) $\frac{r}{\sqrt{3}+1}$ (2) $\frac{r}{2}$
(3) $\frac{r}{\sqrt{3}-1}$ (4) $\frac{2r}{3}$

Answer (1)

Sol. $\frac{kq}{x^2} = \frac{k3q}{(r-x)^2}$

$$x = \frac{r-x}{\sqrt{3}}$$

$$x = \frac{r}{(\sqrt{3}+1)}$$

13. The refractive index of thin prism of an apex angle A is $\cot\left(\frac{A}{2}\right)$. Then the minimum angle of deviation is:

- (1) $180^\circ - 3A$ (2) $180^\circ - 2A$
(3) $180^\circ - A$ (4) $180^\circ - 4A$

Answer (2)

Sol. $\delta_{\min} = 2 \sin^{-1} \left[\mu \sin \frac{A}{2} \right] - A$

$$= 2 \sin^{-1} \left[\cos \frac{A}{2} \right] - A$$

$$= \pi - 2A$$

14. In single electron atom/ion, first member of Lyman series is λ , then wavelength of second member of this series shall be

- (1) $\frac{5}{27} \lambda$ (2) $\frac{5}{32} \lambda$
(3) $\frac{27}{32} \lambda$ (4) $\frac{15}{23} \lambda$

Answer (3)

Sol. $\frac{1}{\lambda} = c \left\{ 1 - \frac{1}{4} \right\}$ for $2 \rightarrow 1$

$$\frac{1}{\lambda'} = c \left\{ 1 - \frac{1}{9} \right\}$$
 for $3 \rightarrow 1$

$$\frac{\lambda'}{\lambda} = \frac{3}{4} \times \frac{9}{8} = \frac{27}{32}$$

15. When light of wavelength λ is incident on a metal, the stopping potential is 8 V. If the wavelength is made 3λ , the stopping potential becomes 2 V. Find the threshold wavelength for the photoelectric effect.

- (1) 2.6×10^{-6} m
- (2) 2.8×10^{-7} m
- (3) 1.24×10^{-6} m
- (4) 1.24×10^{-7} m

Answer (3)

Sol. $(8e) = \frac{hc}{\lambda} - \phi_0 \quad \dots(i)$

$(2e) = \frac{hc}{3\lambda} - \phi_0 \quad \dots(ii)$

$\phi_0 = 1 \text{ eV}$

$\therefore \lambda_{th} = 12400 \text{ \AA}$

$\lambda_{th} = 1.24 \times 10^{-6} \text{ m}$

16. In YDSE, intensity at two sources are in ratio of 1 : 9. If sources are coherent, then intensity at central point is I_1 and if sources are incoherent (and phase differs by 60°), then intensity at central point is I_2 , then $\frac{I_1}{I_2}$ is

- (1) $\frac{10}{13}$
- (2) $\frac{5}{13}$
- (3) $\frac{8}{13}$
- (4) $\frac{7}{11}$

Answer (1)

Sol. $I_1 = 1 + 9 = 10$

$I_2 = 1 + 9 + 2\sqrt{9} \cos 60^\circ = 13$

$\frac{I_1}{I_2} = \frac{10}{13}$

17. Calculate the average energy density of an electromagnetic wave whose electric field is oscillating with amplitude 50 v/m and frequency 5×10^{10} Hz.

- (1) $2 \times 10^{-6} \text{ J/m}^3$
- (2) $1.1 \times 10^{-8} \text{ J/m}^3$
- (3) $3 \times 10^{-7} \text{ J/m}^3$
- (4) $1.6 \times 10^{-7} \text{ J/m}^3$

Answer (2)

Sol. Average energy density = $\frac{1}{2} \epsilon_0 E_0^2$

= $\frac{1}{2} \times 8.85 \times 10^{-12} \times 2500$

= $1.106 \times 10^{-8} \text{ J/m}^3$

- 18.
- 19.
- 20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. A ball dropped from height H rebounds upto height h after colliding with horizontal surface. If coefficient of restitution for collision is $e = \frac{1}{2}$ then $\frac{H}{h}$ shall be

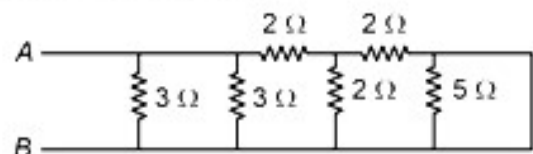
Answer (4)

Sol. Defn of $e = \frac{1}{2} = \frac{\sqrt{2gh}}{\sqrt{2gH}}$

$\frac{1}{4} = \frac{h}{H}$

$\Rightarrow 4$

22. Find equivalent resistance between A and B for given circuit in ohms.



Answer (1)

Sol. \Rightarrow 5 is short circuited

\Rightarrow 2, 2 are parallel \Rightarrow 1

\Rightarrow 1 & 2 in series = 3

$$R_{eq} = \frac{1}{3}(3) = 1\Omega$$

23. A uniform disk of mass 50 kg is rolling with speed of 0.4 m/s. Find minimum energy (in J) required to bring the disk to rest.

Answer (6)

$$\text{Sol. } KE = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\omega^2$$

$$K = \frac{3}{4}mv^2$$

$$= \frac{3}{4} \times 50 \times (4) \times (0.4)$$

$$= 6 \text{ J}$$

24. Mass defect in a nuclear reaction is 0.4 U. The Q value of the reaction is $\frac{x}{10}$ MeV, find x. Take 1 U = 930.5 MeV/c²

Answer (372.2)

$$\text{Sol. } \phi = [930.5 \times 0.4 \text{ MeV/c}^2] \times c^2$$

$$= 372.2 \text{ MeV}$$

25. At any instant, magnetic field inside a coil is 3000 T and it changes 2000 T in next 2 second. If average induced emf through coil is 22 Volt, then find number of turns of coil. (Area of turn is $2 \times 10^{-3} \text{ m}^2$)

Answer (22)

Sol. From Faraday's law

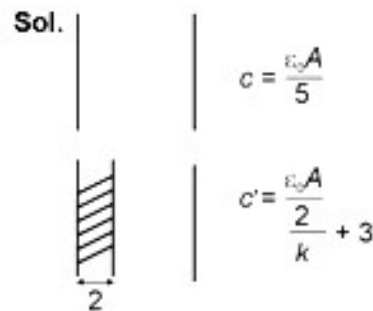
$$\text{Induced emf (e)} = -N \frac{d\Phi}{dt}$$

$$|22| = N \left| \frac{1000}{2} \right| \times 2 \times 10^{-3}$$

$$N = 22$$

26. A parallel plate capacitor with plates separated by 5 mm then it draws current of I_0 from AC source. Now a dielectric of thickness 2 mm is inserted between plates then current drawn increases by 25%. Find dielectric constant.

Answer (2)



$$i = \epsilon_0 \omega C$$

$$I = \epsilon \omega C'$$

$$\Rightarrow \frac{1}{1.25} = \frac{\frac{2}{k} + 3}{5}$$

$$\Rightarrow 5 = \frac{2.5}{k} + 3.75$$

$$\Rightarrow 1.25 = \frac{2.5}{k}$$

$$\Rightarrow k = 2$$

27.

28.

29.

30.

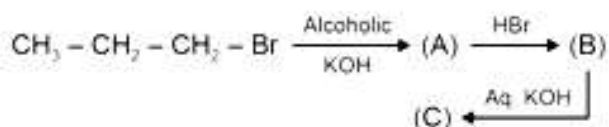
CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

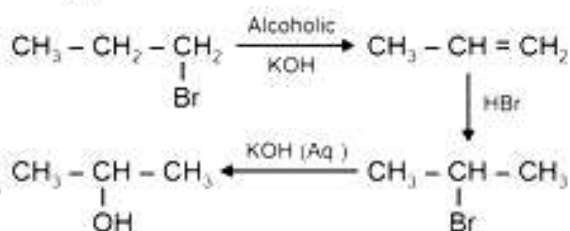
Choose the correct answer :

1. Find out final product (C).



- (1) Propan-1-ol
- (2) Propan-2-ol
- (3) Propene
- (4) Propane

Answer (2)



Sol. $\text{CH}_3 - \underset{\text{OH}}{\text{CH}} - \text{CH}_3$
(C)
Propan-2-ol

2. Which of the following option contain amphoteric oxide(s) only?

- (1) SnO_2 and SiO
- (2) SiO_2
- (3) SnO_2 and PbO_2
- (4) CO and SiO

Answer (3)

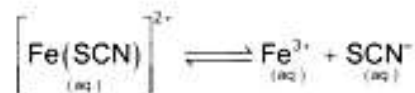
Sol. SnO_2 and PbO_2 are amphoteric oxide.

SiO is weakly acidic oxide.

SiO_2 is acidic oxide.

CO is neutral oxide.

3. Consider the following reaction, the expression for K_c is :



- (1) $K_c = \frac{[\text{Fe}^{3+}][\text{SCN}^-]}{[\text{Fe}(\text{SCN})]^{2+}}$
- (2) $K_c = \frac{[\text{Fe}^{3+}]^2[\text{SCN}^-]}{[\text{Fe}(\text{SCN})]^{2+}}$
- (3) $K_c = \frac{[\text{Fe}^{3+}][\text{SCN}^-]^2}{[\text{Fe}(\text{SCN})]^{2+}}$
- (4) $K_c = \frac{[\text{Fe}^{3+}]^2[\text{SCN}^-]}{[\text{Fe}(\text{SCN})]^{2+}}$

Answer (1)

Sol. $K_c = \frac{\text{Product ion conc.}}{\text{Reactant ion conc.}} = \frac{[\text{Fe}^{3+}][\text{SCN}^-]}{[\text{Fe}(\text{SCN})]^{2+}}$

Hence, option (1) is correct

4. On which factor, electrical conductivity of electrolytic cell doesn't depend

- (1) Concentration of electrolyte
- (2) Nature of electrolyte added
- (3) Temperature
- (4) Nature of electrode

Answer (4)

Sol. Conductivity of electrolytic cell is affected by Conc. of electrolyte, nature of electrolyte and temperature

5. Decreasing order of electron gain enthalpy of the following elements (magnitude only).

- Sulphur → A
Bromine → B
Fluorine → C
Argon → D

- (1) $A > B > C > D$ (2) $D > C > B > A$
 (3) $C > B > A > D$ (4) $A > B > D > C$

Answer (3)

Sol. Electron gain enthalpy values in kJ/mol

$$S = -200$$

$$Br = -325$$

$$F = -333$$

$$Ar = 96$$

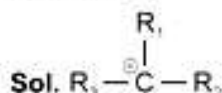
The correct answer is $C > B > A > D$

(Magnitude only) $333 > 325 > 200 > 96$

6. Species having carbon with sextet of valence electrons and acts as an electrophile is called

- (1) Carbanion (2) Carbocation
 (3) Free radical (4) Nitrene

Answer (2)



Carbon has 6 electrons in valence shell.

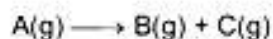
7. The compound which is white in colour is

- (1) $ZnSO_4$ (2) $CuSO_4$
 (3) $FeSO_4$ (4) $FeCl_3$

Answer (1)

Sol. $Zn^{2+} : 4s^0 3d^{10}$ (Colourless)

8. Find the rate constant for first order gaseous reaction.



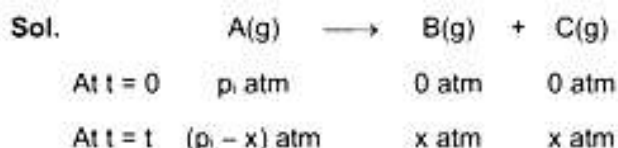
$$(1) k = \frac{2.303}{t} \log \frac{p_i}{2p_i - p_t}$$

$$(2) k = \frac{2.303}{t} \log \frac{2p_i}{p_i - p_t}$$

$$(3) k = \frac{2.303}{t} \log \frac{p_i - p_t}{2p_i}$$

$$(4) k = \frac{2.303}{t} \log \frac{2p_i}{2p_i - p_t}$$

Answer (1)



where p_i is initial pressure

$$p_t = p_i - x + x + x$$

$$p_t = p_i + x$$

$$x = p_t - p_i$$

$$k = \frac{2.303}{t} \log \frac{p_i}{p_i - x}$$

$$k = \frac{2.303}{t} \log \frac{p_i}{p_i - (p_t - p_i)}$$

$$k = \frac{2.303}{t} \log \frac{p_i}{2p_i - p_t}$$

9. **Assertion :** pK_a value of phenol is 10.0 while that of ethanol is 15.9

Reason : Ethanol is stronger acid than phenol

- (1) Both assertion and reason are correct and reason is the correct explanation for assertion
 (2) Assertion is correct and reason is incorrect
 (3) Both assertion and reason are correct but reason is not correct explanation for assertion
 (4) Both assertion and reason are incorrect

Answer (2)

Sol. Since pK_a of phenol is less than ethanol, phenol is the stronger acid.

10. Which of the following solution shows positive deviation from Raoult's law?

- (1) $CHCl_3 + C_6H_6$
 (2) $CH_3COCH_3 + CS_2$
 (3) $CH_3COCH_3 + CHCl_3$
 (4) $CH_3COCH_3 + C_6H_5NH_2$

Answer (2)

Sol. Acetone and CS_2 is an example of solutions showing positive deviation from Raoult's law. Since acetone - CS_2 attractions are weaker than acetone-acetone attractions.

11. **Assertion (A):** Noble gases have very high boiling point.

Reason (R): Noble gases have strong dispersion forces hence they liquify at low temperature, hence they have high boiling point.

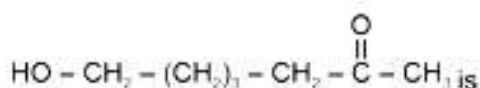
- (1) (A) and (R) are true and (R) explains (A)
 (2) (A) and (R) are true and (R) does not explain (A)
 (3) (A) and (R) are false
 (4) (A) is true but (R) is false

Answer (3)

Sol. Noble gases have very low boiling point due to weak van der Waals forces of attraction. Noble gases do not have interatomic forces other than weak dispersion forces.

Therefore, both (A) and (R) are false.

12. **Statement-I:** IUPAC name of compound

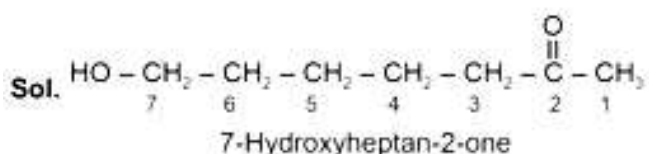


7-hydroxyheptan-2-one.

Statement-II: In IUPAC name -OH is taken as main functional group.

- (1) Both statements I and II are correct
 (2) Both statements I and II are incorrect
 (3) Statement-I is correct, statement-II is incorrect
 (4) Statement-I is incorrect, statement-II is correct

Answer (3)



13. Adsorption principle is used in

- (1) Distillation
 (2) Differential extraction
 (3) Chromatography
 (4) Vacuum distillation

Answer (3)

Sol. Adsorption principle is used in chromatography in which different compounds are adsorbed on an adsorbent to different degrees.

14.
15.
16.
17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

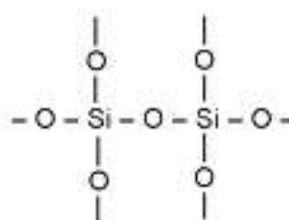
21. How many of the following compounds have sp^3 hybridized central atom?



Answer (3)

Sol. $\text{H}_2\text{O}, \text{NH}_3, \text{SiO}_2$ have sp^3 hybridized central atom.

Structure of SiO_2 is

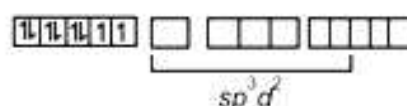


22. The spin only magnetic moment of complex ion $[\text{Ni}(\text{NH}_3)_6]^{2+}$ is $x \times 10^{-1}$ BM

The value of x is _____ (Nearest integer)

Answer (28)

Sol. NH_3 acts as WFL with Ni^{+2} and hybridisation of complex $[\text{Ni}(\text{NH}_3)_6]^{2+}$ is sp^3d^2



No. of unpaired electron = 2

$$\mu = \sqrt{n(n+2)} = \sqrt{2(4)} = \sqrt{8}$$

$$= 2.82$$

$$= 28.2 \times 10^{-1} \text{ BM} \Rightarrow x = 28$$

23. If one Faraday of electricity is used in the discharging of Cu^{2+} , then find the mass (in g) of Cu deposited (Nearest integer)

Answer (32)

Sol. $\text{Cu}^{2+} + 2\text{e}^- \rightarrow \text{Cu}$

Since one mol Cu is deposited by 2 mol or say 2 Faraday charge hence the mol of Cu deposited will be 0.5 mol

$$\text{Mass of Cu deposited} = 63.5 \text{ g mol}^{-1} \times 0.5 \text{ mol}$$

$$= 31.75 \text{ g} = 32 \text{ g}$$

24. The total number of different alkanes formed when the following mixture is subjected to electrolysis: $\text{CH}_3\text{COONa}(\text{aq.})$ and $\text{C}_2\text{H}_5\text{COONa}(\text{aq.})$ (do not consider disproportionation reaction).

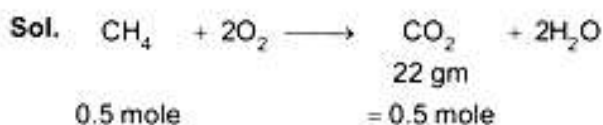
Answer (03.00)

Sol. Ethane, butane and propane are formed when CH_3COONa and $\text{C}_2\text{H}_5\text{COONa}$ undergo electrolysis.

25. Moles of CH_4 required for formation of 22 gm of CO_2 is $m \times 10^{-2}$.

The value of m is

Answer (50)



$$m \times 10^{-2} = 0.5$$

$$m = 50$$

26. How many of the following compounds have sp^3 hybridized central atom?

$\text{BF}_3, \text{BeCl}_2, \text{NH}_3, \text{CH}_4, \text{H}_2\text{O}, \text{SO}_2, \text{CO}_2$

Answer (3)

Sol. $\text{NH}_3, \text{CH}_4, \text{H}_2\text{O}$ have sp^3 hybridized central atoms.

27. How many of the following can be used as electrode in batteries?

- (i) Zinc
- (ii) Zinc-Mercury amalgam
- (iii) Lead
- (iv) Graphite

Answer (4)

Sol. All 4 given species can be used as electrode in batteries.

28. If the energy of radiation having wavelength of 242 nm is $x \times 10^{-19} \text{ J}$, then find nearest integer value of x.

Given : Plank's constant = $6.6 \times 10^{-34} \text{ Joule-Second}$, $c = 3 \times 10^8 \text{ ms}^{-1}$

Answer (8)

Sol. $E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{242 \times 10^{-9}} \text{ Joule}$

$$= \frac{19.8 \times 10^{-26}}{242 \times 10^{-9}}$$

$$= 8.2 \times 10^{-19} \text{ Joule}$$

$$= 8 \times 10^{-19} \text{ Joule}$$

29. How many of the following statements are true?

- (i) Chromate ion is square planar.
- (ii) Green manganate ion is diamagnetic.
- (iii) Dichromate can be prepared using chromate.
- (iv) Dark green KMnO_4 disproportionate in acidic medium and neutral medium.
- (v) For d-block elements, ionic character decreases for increasing oxidation no. of metal in oxides.

Answer (02)

Sol. (iii) and (v) are correct.

The green manganate ion is paramagnetic with one unpaired e^- but the permanganate is diamagnetic



(Ref : NCERT)

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Sum of series $\frac{1}{1-3 \cdot 1^2+1^4} + \frac{2}{1-3 \cdot 2^2+2^4}$
 $+ \frac{3}{1-3 \cdot 3^2+3^4} + \dots$ upto 10 terms is
- (1) $\frac{-55}{109}$ (2) $\frac{55}{109}$
 (3) $\frac{45}{109}$ (4) $\frac{-45}{109}$

Answer (1)

Sol. General term of series

$$\Rightarrow T_r = \frac{r}{1-3r^2+r^4}$$

$$= \frac{r}{(r^4-2r^2+1)-r^2} = \frac{r}{(r^2-1)^2-r^2}$$

$$= \frac{r}{(r^2-r-1)(r^2+r-1)} = \frac{1}{2} \left[\frac{1}{r^2-r-1} - \frac{1}{r^2+r-1} \right]$$

Sum of 10 terms \Rightarrow

$$\sum_{r=1}^{10} T_r = \frac{1}{2} \left[\frac{1}{-1} - \frac{1}{1} \right] + \frac{1}{2} \left[\frac{1}{1} - \frac{1}{5} \right] + \frac{1}{2} \left[\frac{1}{5} - \frac{1}{11} \right] + \dots$$

$$+ \frac{1}{2} \left[\frac{1}{89} - \frac{1}{109} \right]$$

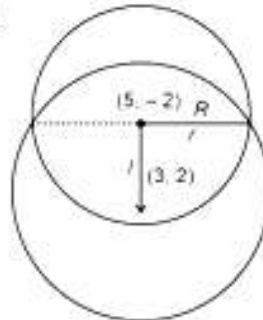
Telescopic sum

$$\Rightarrow \frac{1}{2} \left[-1 - \frac{1}{109} \right] = \frac{1}{2} \left(\frac{-110}{109} \right) = \frac{-55}{109}$$

2. If one of the diameter of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle and whose centre is the point of intersection of the lines $2x + 3y = 12$ and $3x - 2y = 5$, then the radius of the circle is
- (1) 6 (2) $3\sqrt{2}$
 (3) $\sqrt{20}$ (4) 4

Answer (1)

Sol.



$$2x + 3y = 12$$

$$3x - 2y = 5$$

Point of intersection = (3, 2)

$$\therefore \text{centre is } (3, 2)$$

$$l = \sqrt{4 + 16} = 2\sqrt{5}$$

$$P + R^2 = r^2$$

$$\Rightarrow 20 + (25 + 4 - 13) = r^2$$

$$\Rightarrow 20 + 16 = r^2$$

$$\Rightarrow r = 6$$

3. An urn contains 15 red, 10 white, 60 orange and 15 green balls. 2 balls are taken with replacement. Find the probability 1 ball is red and other ball is white.
- (1) $\frac{2}{27}$ (2) $\frac{3}{22}$
 (3) $\frac{1}{33}$ (4) $\frac{1}{29}$

Answer (3)

Sol. Total balls in urn = 15 + 10 + 60 + 15 = 100 balls

2 balls are taken with replacement
 So, probability that 1 ball is red and

$$1 \text{ ball is white} = \frac{{}^{15}C_1 \times {}^{10}C_1}{{}^{100}C_2}$$

$$\frac{15 \times 10 \times 2}{100 \times 99} = \frac{300}{100 \times 99} = \frac{3}{99} = \frac{1}{33}$$

4. $\lim_{x \rightarrow 0} \frac{e^{2 \sin x} - 2|\sin x| - 1}{x^2}$ is
- (1) D.N.E (2) 2
 (3) 1 (4) -1

Answer (2)

$$= \frac{-2}{3} \ln\left(\frac{2}{3}\right) + \frac{1}{9}$$

$$\Rightarrow f(-1) = -p = \frac{-2}{3} \ln\left(\frac{2}{3}\right) + \frac{1}{9}$$

$$\Rightarrow p = \frac{2}{3} \ln\left(\frac{2}{3}\right) - \frac{1}{9}$$

$$g(3) = 3p = 2 \ln\left(\frac{2}{3}\right) - \frac{1}{3} = \ln\left(\frac{4}{9}\right) + \log e^{(-\frac{1}{3})}$$

$$\Rightarrow \ln\left(\frac{4}{9e^{\frac{1}{3}}}\right)$$

12. The distance of the point $Q(0, 2, -2)$ from the line passing through the $P(5, -4, 3)$ and perpendicular to the lines $\vec{r} = (-3\hat{i} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 5\hat{k}), \lambda \in \mathbb{R}$ and $\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \mu(-\hat{i} + 3\hat{j} + 2\hat{k}), \mu \in \mathbb{R}$ is

(1) $\sqrt{66}$

(2) $\sqrt{74}$

(3) $\sqrt{56}$

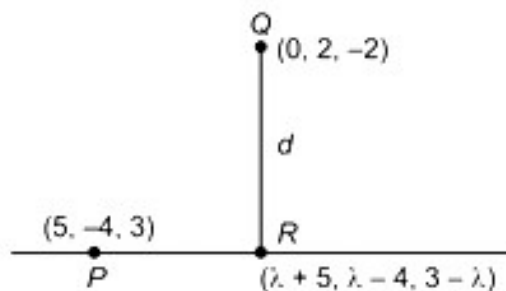
(4) $\sqrt{46}$

Answer (2)

Sol. $\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix} = \hat{i}(6-15) - \hat{j}(4+5) + \hat{k}(6+3)$
 $= -9\hat{i} - 9\hat{j} + 9\hat{k}$

\therefore Required line (L): $\frac{x-5}{1} = \frac{y+4}{1} = \frac{z-3}{-1} = \lambda$

$x = \lambda + 5, y = \lambda - 4, z = 3 - \lambda$



$\vec{QR} \cdot \vec{c} = (\lambda+5) \cdot 1 + (\lambda-6) \cdot 1 + (3-\lambda+2) \cdot (-1) = 0$

$\Rightarrow \lambda + 5 + \lambda - 6 - 5 + \lambda = 0$

$\Rightarrow 3\lambda = 6$

$\Rightarrow \lambda = 2$

$\therefore R(7, -2, 1)$

$\therefore QR = \sqrt{49 + 16 + 9}$

$= \sqrt{74}$

13. If α denotes the number of real solutions of $(1-i)^x = 2^x$ and $\beta = \frac{|z|}{\arg z}$,

where $z = \frac{\pi}{4}(1+i)^4 \left[\frac{1-\sqrt{\pi i}}{\sqrt{\pi+i}} + \frac{\sqrt{\pi-i}}{1+\sqrt{\pi i}} \right], i = \sqrt{-1}$

then distance of (α, β) from the line $4x - 3y - 7 = 0$ is

(1) 2

(2) 3

(3) 7

(4) 4

Answer (2)

Sol. $z = \frac{\pi}{4}(1+i)^4 \left[\frac{1+\pi+\pi+1}{i(1+\pi)-\sqrt{\pi}+\sqrt{\pi}} \right]$

$= \frac{\pi}{4}(1+i)^4 \frac{2}{i} = \frac{\pi(1+i)^4}{2i} = \frac{-\pi}{2}i(1+i)^4$

$|z| = \left| -\frac{\pi}{2}i \right| |1+i|^4 = \left(\frac{\pi}{2}\right)(1)(\sqrt{2})^4 = 2\pi$

$z = \frac{-\pi}{2}(-4i) = 2\pi i \Rightarrow \arg(z) = \frac{\pi}{2}$

$\Rightarrow \beta = \frac{2\pi}{\frac{\pi}{2}} = 4$

Also, $(1-i)^x = 2^x$

\Rightarrow Taking modulus both sides

$\Rightarrow (\sqrt{2})^x = 2^x \Rightarrow (\sqrt{2})^x = 0$

\Rightarrow at $x = 0$

\Rightarrow 1 solution $\Rightarrow \alpha = 1$

\Rightarrow Perpendicular distance from $(1, 4)$

$\left| \frac{4(1) - 3(4) - 7}{5} \right| = \left| \frac{4 - 12 - 7}{5} \right| = 3$

14. Let S be the set of positive integral values of 'a' for which $\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32} < 0 \quad \forall x \in \mathbb{R}$

Then the number of elements in S is

(1) 1

(2) 3

(3) 0

(4) ∞

Answer (3)

Sol. Given $\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32} < 0 \quad \forall x \in \mathbb{R}$

In $x^2 - 8x + 32$, we have $D = 64 - 128 < 0$

$\therefore x^2 - 8x + 32 > 0 \quad \forall x \in \mathbb{R}$

$$\Rightarrow ax^2 + 2(a+1)x + 9a + 4 < 0 \quad \forall x \in \mathbb{R}$$

$$\Rightarrow a < 0 \text{ and } D < 0$$

\therefore Question has asked for positive integral values of a

$$\therefore |S| = 0$$

15. For $\alpha, \beta, \gamma > 0$, if $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$, then γ is

$$(1) -\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) \quad (2) \frac{-1}{\sqrt{2}}$$

$$(3) -\sqrt{3} \quad (4) \frac{\sqrt{3}}{2}$$

Answer (4)

Sol. Let $\sin A = \alpha$

$$\sin B = \beta$$

$$\sin C = \gamma$$

$$A + B + C = \pi$$

$$\Rightarrow (\alpha + \beta)^2 - \gamma^2 = 3\alpha\beta$$

$$\Rightarrow \alpha^2 + \beta^2 - \alpha\beta = \gamma^2$$

$$\Rightarrow \alpha^2 + \beta^2 - \gamma^2 = \alpha\beta$$

$$\Rightarrow \frac{\alpha^2 + \beta^2 - \gamma^2}{2\alpha\beta} = \frac{1}{2}$$

$$\Rightarrow \cos C = \frac{1}{2} \Rightarrow C = 60^\circ$$

$$\Rightarrow \sin C = \frac{\sqrt{3}}{2} = \gamma$$

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In the expansion of

$$(1+x)(1-x^2)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5, \text{ the sum of coefficients of } x^3 \text{ and } x^{-13} \text{ is}$$

Answer (118)

$$\text{Sol. } (1+x)(1-x^2)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$$

$$\Rightarrow (1+x)^2(1-x)\frac{(x^3+3x^2+3x+1)^5}{x^{15}}$$

$$\Rightarrow \frac{(1+x)^2(1-x)[(x+1)^3]^5}{x^{15}} = \frac{(1-x)(1+x)^{17}}{x^{15}}$$

$$\Rightarrow \text{Coefficient of } x^3 \Rightarrow x^{18} \text{ in } (1-x)(1+x)^{17}$$

$$\Rightarrow (1+x)^{17} - x(1+x)^{17}$$

$$\Rightarrow 0 - x\left({}^{17}C_{17} x^{17}\right) = -{}^{17}C_{17} = -1$$

$$\text{and coefficient of } x^{-13} \Rightarrow x^2 \text{ in } (1-x)(1+x)^{17}$$

$$\Rightarrow (1+x)^{17} - x(1+x)^{17}$$

$$\Rightarrow {}^{17}C_2 - {}^{17}C_1 = 17 \times 8 - 17$$

$$= 17 \times 7 = 119$$

$$\Rightarrow \text{sum} = 119 - 1 = 118$$

22. $A = \{1, 2, 3, 4\}$, $R = \{(1, 2), (2, 3), (2, 4)\}$ $R \subseteq S$ and S is an equivalence relation, then minimum number of elements to be added to R is n then value of n ?

Answer (13)

Sol. $R = \{(1, 2), (2, 3), (2, 4)\}$

for reflexive, we need to add,

$(1, 1), (2, 2), (3, 3), (4, 4)$

for symmetric

if $(1, 2) \in R$

then $(2, 1) \in R$

if $(2, 3) \in R$

then $(3, 2) \in R$

if $(2, 4) \in R$

then $(4, 2) \in R$

So set becomes

$\{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (2, 3), (3, 2), (2, 4), (4, 2)\}$

for transitive

As $(1, 2) \in R$

$(2, 3) \in R$

then $(1, 3) \in R$ then $(3, 1) \in R$ (for symmetric)

& $(1, 2) \in R$

$(2, 4) \in R$

then $(1, 4) \in R$ then $(4, 1) \in R$ (for symmetric)

$$(3, 2) \in R$$

$$(2, 4) \in R$$

then $(3, 4) \in R$ then $(4, 3) \in R$ (for symmetric)

so set $S = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1),$

$(2, 3), (3, 2), (2, 4), (4, 2), (1, 3), (3, 1), (1, 4), (4,$

$1), (3, 4), (4, 3)\}$

so 13 new elements are added

$$\Rightarrow n = 13$$

23. If $|\vec{a}| = 1, |\vec{b}| = 4$ are $\vec{a} \cdot \vec{b} = 2$

also, $\vec{c} = (3\vec{a} \times \vec{b}) - \vec{b}$ and α is the angle between

\vec{b} and \vec{c} then the value of $192\sin^2\alpha$.

Answer (167)

Sol. $|\vec{c}|^2 = 9(|\vec{a} \times \vec{b}|)^2 + |\vec{b}|^2 + 0$

$$|\vec{c}|^2 = 9(16 - 4) + 16$$

$$\{\because |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2\}$$

$$|\vec{c}|^2 = 124$$

$$|\vec{c}| = \sqrt{124}$$

$$\vec{c} = (3(\vec{a} \times \vec{b})) - \vec{b}$$

$$\vec{c} \cdot \vec{b} = -|\vec{b}|^2 = -16$$

$$4 \times \sqrt{124} \cos\alpha = -16$$

$$\cos\alpha = \frac{-4}{\sqrt{124}} = \frac{-2}{\sqrt{31}}$$

$$\sin\alpha = \sqrt{1 - \frac{4}{31}}$$

$$\sin\alpha = \sqrt{\frac{27}{31}}$$

$$\text{Then, } 192\sin^2\alpha = 192 \times \frac{27}{31}$$

$$\approx 167.2$$

24. If the system of linear equation $x - 2y + z = -4, 2x + \alpha y + 3z = 5$ & $3x - y + \beta z = 3$ has infinitely many solutions then $12\alpha + 13\beta$ is equal to

Answer (58)

Sol. $x - 2y + z = -4$

$$2x + \alpha y + 3z = 5$$

$$3x - y + \beta z = 3$$

$$\Delta_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$$

$$(5\beta - 9) + 4(2\beta - 9) - 9 = 0$$

$$13\beta = 54$$

$$\beta = \frac{54}{13}$$

$$\Delta_3 = \begin{vmatrix} 1 & -2 & -4 \\ 2 & \alpha & 5 \\ 3 & -1 & 3 \end{vmatrix} = 0$$

$$(3\alpha + 5) + 2(-9) - 4(-2 - 3\alpha) = 0$$

$$3\alpha + 5 - 18 + 8 + 12\alpha = 0$$

$$\Rightarrow \alpha = \frac{1}{3}$$

$$12\alpha + 13\beta = 4 + 13 \times \frac{54}{13}$$

$$= 58$$

25.

26.

27.

28.

29.

30.

