JEE (Main)-2024 : Phase-1 (29-01-2024)-Morning

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. A body of man 100 kg travelled 10 m before coming to rest. If $\mu = 0.4$, work done against friction is (motion is happening on horizontal surface, take $g = 10\ m/s^2$)

(1) 4500 J (2) 5000 J

(3) 4200 J (4) 4000 J

Answer (4)

Sol. $\frac{v^2}{2a} = s$ $(a = \mu g)$

$v^2 = 2 \times \mu g\, s$

$v^2 = 2 \times (.4) \times 10 \times 10$

$v^2 = 80$

$w_f = \Delta k$

$= -\frac{1}{2} \times 100 \times 80$

$w_f = -4000$

2. If an object is having same weight at same distance above and below the surface of earth, find its distance from surface of earth.

(1) $\frac{R}{2}$ (2) $\left(\sqrt{5}-1\right)\frac{R}{2}$

(3) $\left(\sqrt{3}-1\right)\frac{R}{2}$ (4) $\left(\sqrt{5}-1\right)R$

Answer (2)

Sol. $\frac{GMm}{(R+x)^2} = \frac{GMm\,(R-x)}{R^3}$

$\Rightarrow R^3 = (R+x)^2\,(R-x)$

$\Rightarrow R^3 = (R^2 - x^2)\,(R+x)$

$\Rightarrow x^2 + Rx - R^2 = 0$

$\therefore x = \frac{-R \pm \sqrt{R + 4R^2}}{2}$

$$\boxed{x = \frac{\left(\sqrt{5}-1\right)}{2}R}$$

3. Consider the two statements :

Statement-1 : A capillary tube is first dipped in hot water and then dipped in cold water. The rise is higher in hot water.

Statement-2 : Capillary tube is first dipped in cold water and then dipped in hot water. The rise is higher in cold water.

(1) Statement-1 is true and statement-2 is false

(2) Statement-1 is false and statement-2 is true

(3) Both statements are true

(4) Both statements are false

Answer (2)

Sol. $h = \frac{2S\cos\theta}{\rho g R}$

as $T\uparrow$, $S\downarrow$

The correct answer is Option (2).

4. If a particle starting from rest having constant acceleration covers distance S_1 in first $(p-1)$ seconds and S_2 in first p seconds, then determine time for which displacement is $S_1 + S_2$

(1) $\sqrt{2p^2 + 1 - 2p}$

(2) $\sqrt{2p^2 + 1 + 2p}$

(3) $\sqrt{(p-1)^2 - p}$

(4) $2p$

Answer (1)

Sol. $S_1 = \frac{1}{2}a(p-1)^2$

$S_2 = \frac{1}{2}ap^2$

$S_1 + S_2 = \frac{1}{2}a\left[(p-1)^2 + p^2\right] = \frac{1}{2}at^2$

$t = \sqrt{2p^2 + 1 - 2p}$

5. de-Broglie wavelength of a proton and an electron is same. The ratio of kinetic energy of electron to that of proton is

(1) 1 (2) 1835

(3) $\frac{1}{1867}$ (4) 933.5

Answer (2)

Sol. $\frac{h}{p_1} = \frac{h}{p_2}$

$\Rightarrow \sqrt{2m_1k_1} = \sqrt{2m_2k_2}$

$\Rightarrow \frac{k_2}{k_1} = \frac{m_1}{m_2} = 1835$

6. If ratio of centripetal acceleration of two particles moving on the same path is 3 : 4. Find the ratio of their tangential velocities.

(1) $2:\sqrt{3}$

(2) $\sqrt{3}:2$

(3) $\sqrt{3}:1$

(4) $\sqrt{2}:1$

Answer (2)

Sol. $a_c = \frac{v^2}{r}, \frac{(a_c)_1}{(a_c)_2} = \left(\frac{v_1}{v_2}\right)^2$

$\frac{3}{4} = \left(\frac{v_1}{v_2}\right)^2 \rightarrow \frac{v_1}{v_2} = \sqrt{3}:2$

7. A capacitor having capacitance of 100 μF is charged with a potential difference of 12 V is connected to an inductor of inductance 10 mH. Find the maximum current through the inductor.

(1) 2 A (2) 1.6 A

(3) 2.4 A (4) 1.2 A

Answer (4)

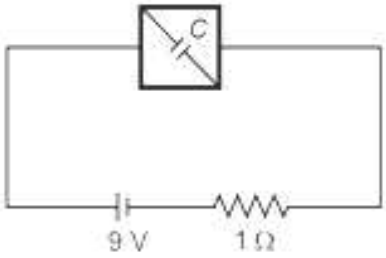
Sol. $I = Q_{\omega}$

$= \frac{CV}{\sqrt{LC}} = V\sqrt{\frac{C}{L}}$

$= 12\sqrt{\frac{100\times10^{-6}}{10\times10^{-3}}}$

$= 1.2$ A

8. A square loop of resistance 16 Ω is connected with battery of 9 V and internal resistance of 1 Ω. In steady state, find energy stored in capacitor of capacity $C = 4$ μF as shown (at steady state current divides symmetrically)



(1) 51.84 μJ

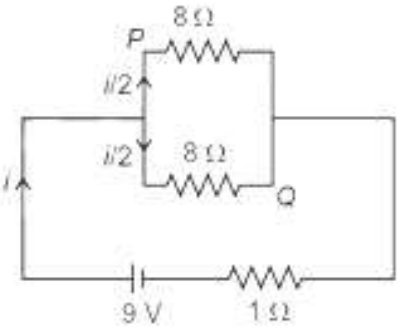
(2) 12.96 μJ

(3) 25.92 μJ

(4) 103.68 μJ

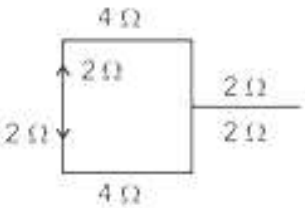
Answer (3)

Sol. Equivalent circuit



$I = \frac{9}{4+1} = 1.8$ A

$\Rightarrow \frac{I}{2} = 0.9$ A



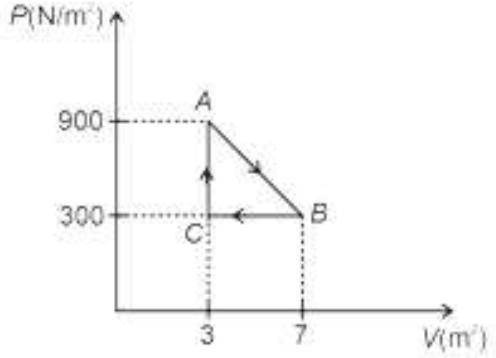
$(V_P - V_Q) = 0.9 \times 6 - 0.9 \times 2$

$V_C = 3.6$ V

$U = \frac{1}{2}CV^2 = \frac{1}{2}\times4\times3.6\times3.6$ μJ

$= 25.92$ μJ

9. A gas undergoes a cyclic process *ABCA* as shown. Find the work done by the gas for $A \longrightarrow B \longrightarrow C$.



(1) 1800 J

(2) 1200 J

(3) 3600 J

(4) 600 J

Answer (2)

Sol. Work = Area

$$\Rightarrow W = \frac{1}{2} \times 600 \times 4$$

$$= 1200 \text{ J}$$

10. If a biconvex lens of material of refractive index 1.5 has focal length 20 cm in air, then its focal length when it is submerged in a medium of refractive index 1.6 is

(1) –160 cm

(2) 160 cm

(3) 1.6 cm

(4) – 16 cm

Answer (1)

Sol. $\frac{1}{20} = (1.5 - 1)\left(\frac{2}{R}\right)$

$R = 20$ cm

$$\frac{1}{f'} = \left(\frac{1.5}{1.6} - 1\right)\left(\frac{2}{R}\right)$$

$$= \frac{-1}{16} \times \frac{2}{20}$$

$\boxed{f' = -160 \text{ cm}}$

11. If electric current passing through a conductor varies with time as $I = I_0 + \beta t$, where $I_0 = 20$ A, $\beta = 3$ A/s, then find charge flow through conductor in first 10 sec.

(1) 400 C (2) 500 C

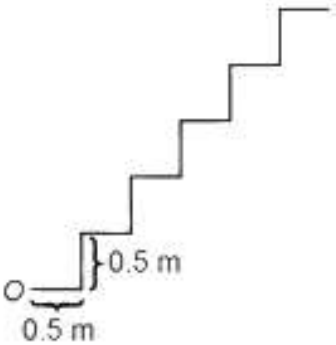
(3) 200 C (4) 350 C

Answer (4)

Sol. $\Rightarrow d = \int I.dt = \int_0^{10} (20 + 3t)dt$

$$= (20t)_0^{10} + 3\left(\frac{t^2}{2}\right)_0^{10} = 350 \text{ C}$$

12. Consider a series of steps as shown. A ball is thrown from *O*. Find the minimum speed of directly jump to 5th step.



(1) $5\left(\sqrt{2}+1\right)$ m/s (2) $5\sqrt{2}$ m/s

(3) $5\sqrt{\sqrt{2}+1}$ m/s (4) $6\sqrt{\sqrt{3}+1}$ m/s

Answer (3)

Sol. $y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$

(2.5, 2.5) must lie on this

$$\Rightarrow 1 = \tan\theta - \frac{g \times 2.5}{2v^2\cos^2\theta}$$

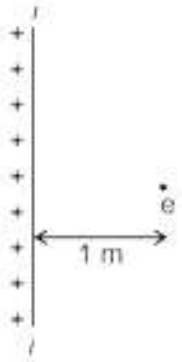
$$\Rightarrow \frac{25}{2v^2\cos^2\theta} = \tan\theta - 1$$

$$\Rightarrow v^2 = \frac{25}{2}\left\{\frac{1+\tan^2\theta}{\tan\theta - 1}\right\}$$

$$\Rightarrow v_{min} = 5\sqrt{\sqrt{2}+1}$$

[Happens when $\tan\theta = \sqrt{2}+1$]

13. An electron is moving with speed of 1 m/s at distance of 1 m from a large sheet of charge with density σ C/m^2. Find maximum value of σ such that electron hit the sheet after 1 sec.



(mass of electron 9×10^{-31} kg, permittivity of free space $\varepsilon_0 = 9 \times 10^{-12}$ C^2/Nm^2)

(1) 4.05×10^{-22} C/m^2

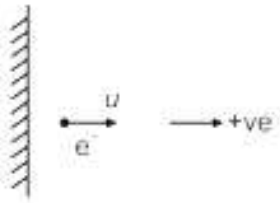
(2) 8.10×10^{-22} C/m^2

(3) 4.05×10^{-24} C/m^2

(4) 2.02×10^{-20} C/m^2

Answer (1)

Sol. For maximum value of σ, initially, electron must move away from plate.



$$ut + \frac{1}{2}at^2 = s$$

$t = 1 \quad u = 1$ m/s $\quad s = -1$ m

$$1 \times 1 - \frac{1}{2}a \times 1^2 = -1$$

$$\Rightarrow \quad a = 4 \text{ m/s}^2$$

$$\frac{qE}{m} = 4$$

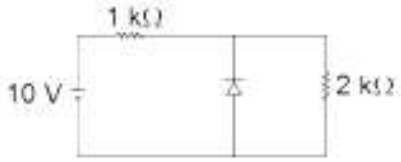
$$\frac{q\sigma}{2\varepsilon_0 m} = 4$$

$$\sigma = \frac{4 \times 2 \times 9 \times 10^{-12} \times 9 \times 10^{-31}}{1.6 \times 10^{-19}}$$

$$= \frac{8 \times 81}{1.6} \times 10^{-24}$$

$$= 4.05 \times 10^{-22} \text{ C/m}^2$$

14. In the voltage regulator circuit shown below, the reverse breakdown voltage of zener diode is 3 V. Find the current through zener diode.



(1) 7 mA

(2) 1.5 mA

(3) 5.5 mA

(4) 10 mA

Answer (3)

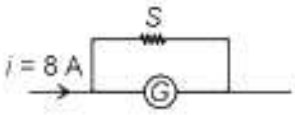
Sol. $i_{\text{battery}} = \frac{10-3}{1000} = 7$ mA

$i_{2k\Omega} = \frac{3}{2000} = 1.5$ mA

$i_2 = (7 - 1.5)$ mA

$= 5.5$ mA

15. Consider the circuit shown. Galvanometer resistance is 10 Ω and current through galvanometer is 3 mA. Find the resistance of shunt.



(1) 10^{-3} Ω

(2) 7.5×10^{-3} Ω

(3) 6.75×10^{-3} Ω

(4) 3.75×10^{-3} Ω

Answer (4)

Sol. Since G and S are in parallel

$\Rightarrow V_G = V_S$

$\Rightarrow 3\text{ mA} \times 10 = 8\text{ A} \times R_S$

$\Rightarrow R_S = 3.75\text{ m}\Omega$

16. A particle executing simple harmonic motion along x-axis, with amplitude A, about origin. If ratio of kinetic energy and total energy at $x = \frac{A}{3}$ is

(1) $\frac{8}{9}$

(2) $\frac{7}{8}$

(3) $\frac{1}{9}$

(4) $\frac{1}{8}$

Answer (1)

Sol. $KE = \frac{1}{2} m\omega^2 (A^2 - n^2)$

$TE = \frac{1}{2} m\omega^2 A^2$

$\frac{KE}{TE} = \frac{A^2 - n^2}{A^2} = \frac{1 - \frac{1}{9}}{1} = \frac{8}{9}$

17.

18.

19.

20.

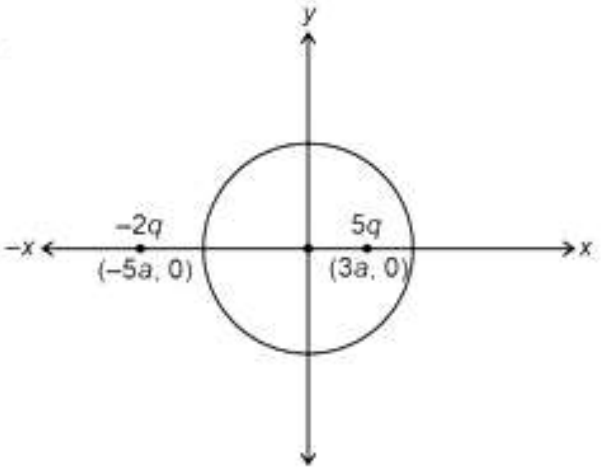
SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. A solid sphere of radius $4a$ with centre at origin. Two charge, $-2q$ at $(-5a, 0)$ and $5q$ at $(3a, 0)$ is placed. Flux through sphere is $\frac{xq}{\varepsilon_0}$. Find x

Answer (5)

Sol.



From Gauss law

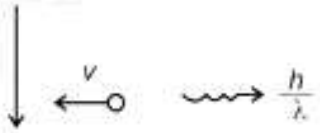
$\phi = \frac{q_{enclosed}}{\varepsilon_0} = \frac{5q}{\varepsilon_0}$

22. A stationary hydrogen atom de excites from first excited state to ground state. Find recoil speed of hydrogen atom up to nearest integral value. (mass of hydrogen atom = 1.8×10^{-27} kg)

Answer (3)

Sol. $|\Delta E_0| = \left(-13.6\left\{1 - \frac{1}{4}\right\}\right)$ ev

$|\Delta E| = 10.2$ ev



$\lambda = \frac{12400}{10.2} \times 10^{-10}$ m

$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \times 10.2}{12400 \times 10^{-10}}$

$\because \quad mv = \frac{h}{\lambda}$

$\therefore \quad 1.8 \times 10^{-27}$

$v = \frac{6.63 \times 10.2 \times 10^{-34}}{12400 \times 10^{-10}}$

$v = \frac{6.63 \times 10.2}{12400 \times 1.8} \times 10^3$

$= \frac{6.63 \times 102}{124 \times 1.8} = 3.02$

$\approx$ 3 m/s

23. In a container, 1 g of hydrogen and 1 g of oxygen are taken. Find the ratio of hydrogen pressure to oxygen pressure.

Answer (16)

Sol. $PV = nRT$

$\Rightarrow P \propto n$

$\Rightarrow$ Ratio $= \frac{32}{2} = 16$

24. In a convex mirror having radius of curvature 30 cm the height of image is half the object height. What will be the object (in cm) distance?

Answer (15)

Sol. $f = 15$

$$m = -\frac{v}{u} = +\frac{1}{2}$$

$$v = -\frac{u}{2}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

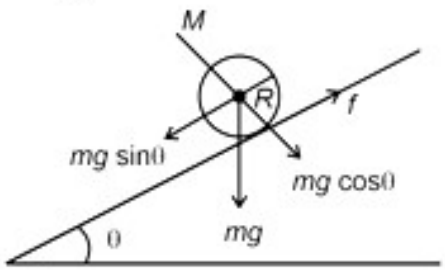
$$\frac{2}{-u} + \frac{1}{u} = \frac{1}{f}$$

$u = -f = -15$ cm

25. A solid cylinder is placed gently over an incline plane of inclination 60°. The acceleration of cylinder when it start rolling without slipping is $\frac{g}{\sqrt{x}}$, where μ is coefficient of friction. (Take $g = 10$ m/s^2)

Answer (3)

Sol.



Since $a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$

$$\Rightarrow a = \frac{g \times \frac{\sqrt{3}}{2}}{1 + \frac{1}{2}} = \frac{g\frac{\sqrt{3}}{2}}{\frac{3}{2}}$$

$$\Rightarrow a = \frac{g}{\sqrt{3}}$$

26. Voltage and resistance for a resistor are measured as $V = 200 \pm 5$ volts and $R = 20 \pm 0.2\ \Omega$. The percentage error in current $I = \frac{V}{R}$ is x. Find the value of $10x$

Answer (35)

Sol. $\%\ \text{error} = \left(\frac{dV}{V} + \frac{dR}{R}\right) \times 100$

$$= \left(\frac{5}{200} + \frac{0.2}{20}\right) \times 100$$

$= 3.5$

27. Potential energy function corresponding to a conservative force is given as $U(x,y,z) = \frac{3x^2}{2} + 5y + 6z$, then the force at $x = 6$ is pN. The value of p upto its nearest integral value is

Answer (20)

Sol. $F_x = \frac{-dv}{dx}$

$\vec{F} = -3x\hat{i} - 5\hat{j} - 6\hat{k}$

$\left|\vec{F}\right|_{x=6} = \sqrt{18^2 + 5^2 + 6^2}$

$= \sqrt{324 + 25 + 36}$

$= \sqrt{385}$

$= 19.62$ N

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Which of the following pair will be formed by the decomposition of $KMnO_4$?

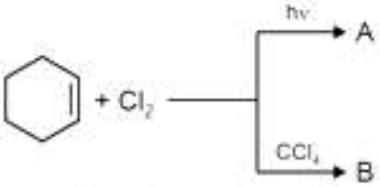
   (1) $KMnO_4, MnO_2$ (2) K_2MnO_4, MnO_2

   (3) K_2MnO_4, H_2O (4) MnO_2, H_2O

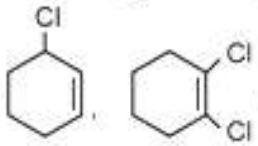
Answer (2)

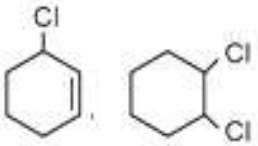
Sol. $KMnO_4$ decomposes upon heating at 513 K and forms K_2MnO_4 and MnO_2.

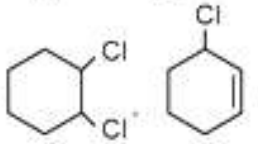
$$2KMnO_4 \xrightarrow{\Delta} K_2MnO_4 + O_2 + MnO_2$$

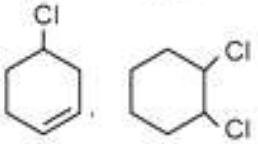
2. 

In the following reactions, find the product A and B?

(1)  (A) (B)

(2)  (A) (B)

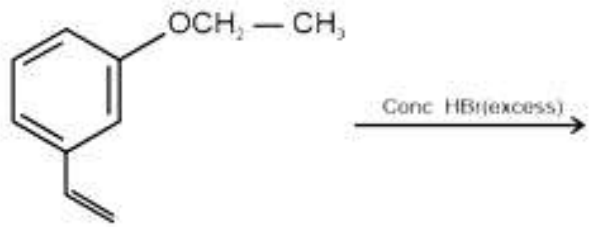
(3)  (A) (B)

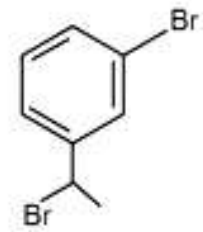
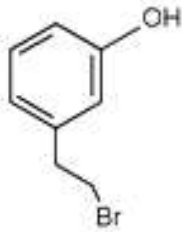
(4)  (A) (B)

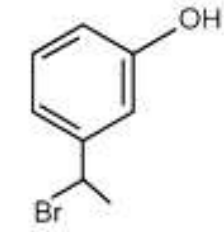
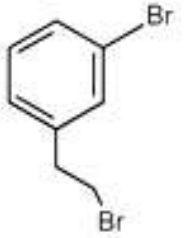
Answer (2)

Sol.
- In presence of light allylic substitution occur.
- In presence of CCl_4, addition reaction will occur.

3. The major product formed in the following reaction is :



(1)  (2) 

(3)  (4) 

Answer (3)

Sol. HBr adds to alkene in accordance with Markovnikov's rule

4. Which of the following coordination compounds has bridging carbonyl ligand?

   (1) $[Mn_2(CO)_{10}]$

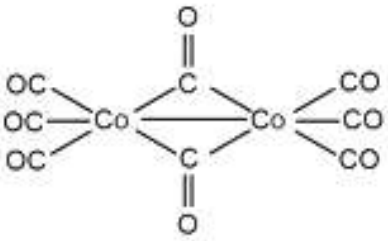
   (2) $[Co_2(CO)_8]$

   (3) $[Cr(CO)_6]$

   (4) $[Fe(CO)_5]$

Answer (2)

Sol.



From structure it is clear $[Co_2(CO)_8]$ has bridging carbonyl ligand.

5. Energy difference between actual structure of compound and most stable resonating structure having least energy is called:

(1) Heat of hydrogenation

(2) Resonance energy

(3) Heat of combustion

(4) Exchange energy

Answer (2)

Sol. Resonance energy is the energy difference between most stable resonating structure and actual structure.

6. What is the effect that occurs between lone pair and π-bond?

(1) Inductive (2) Electromeric

(3) Resonance (4) Hyperconjugation

Answer (3)

Sol. $\overset{\odot}{X}-Y=Z \longleftrightarrow \overset{\oplus}{X}=Y-\overset{\ominus}{Z}$

Resonance

Above effect is called Resonance.

Correct answer is option (3).

7. Which of the following statement is incorrect?

(1) $\Delta G = 0$ for reversible process

(2) $\Delta G < 0$ for spontaneous process

(3) $\Delta G > 0$ for spontaneous process

(4) $\Delta G > 0$ for non-spontaneous process

Answer (3)

Sol. For spontaneous process $\Delta G < 0$

For reversible process $\Delta G = 0$

8. Alkaline $KMnO_4$ oxidises Iodide to a particular product (A). Determine the oxidation state of Iodine in compound (A).

(1) +2 (2) +3

(3) +5 (4) +7

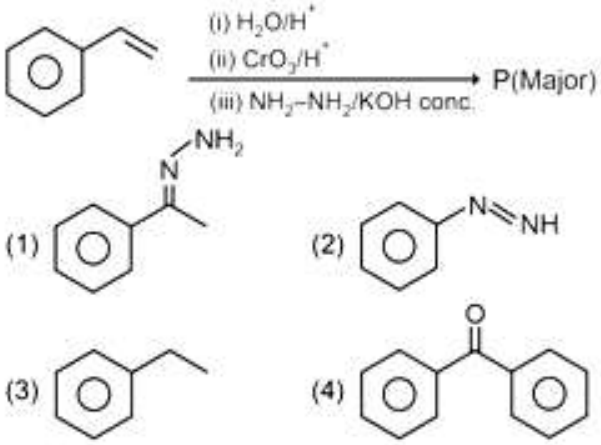
Answer (3)

Sol. Potassium permanganate in alkaline medium oxidise Iodide to Iodate.

$$2MnO_4^- + H_2O + I^\ominus \longrightarrow 2MnO_2 + 2OH^\ominus + \underset{(A)}{IO_3^\ominus}$$

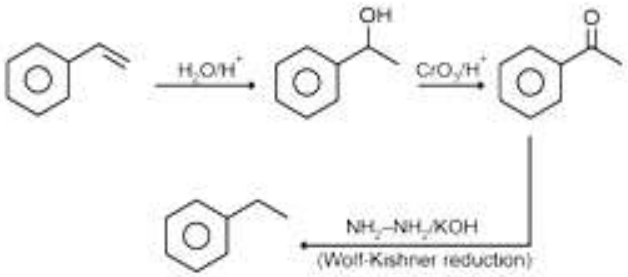
Compound A is $IO_3^\ominus$. Therefore, oxidation state of I is +5.

9. Find product P of the following reaction.



Answer (3)

Sol.



10. A container contains 1 g H_2 gas and 1 g O_2 gas, what is the ratio of partial pressure of H_2 and O_2 $\left(\frac{p_{H_2}}{p_{O_2}}\right)$?

(1) 16 : 1 (2) 8 : 1

(3) 4 : 1 (4) 1 : 1

Answer (1)

Sol. $p_{H_2} = P_T \chi_{H_2}$ (P_T = total pressure)

(χ_{H_2} = mole fraction of H_2)

$p_{O_2} = P_T \chi_{O_2}$ (χ_{O_2} = mole fraction of O_2)

$$\frac{p_{H_2}}{p_{O_2}} = \frac{\chi_{H_2}}{\chi_{O_2}} = \frac{n_{H_2}}{n_{O_2}}$$

$$n_{H_2} = \frac{1}{2} \text{ mol}$$

$$n_{O_2} = \frac{1}{32}$$

$$\frac{p_{H_2}}{p_{O_2}} = \frac{1}{2 \times 1} \times 32$$

$$\frac{p_{H_2}}{p_{O_2}} = \frac{32}{2} = \frac{16}{1}$$

11. Match the following.

	Column I (Ores)		Column II (Formula)
(A)	Fluorspar	(p)	$Al_2O_3.2H_2O$
(B)	Cryolite	(q)	CaF_2
(C)	Bauxite	(r)	$MgCO_3.CaCO_3$
(D)	Dolomite	(s)	$Na_3[AlF_6]$

(1) (A)-(s); (B)-(q); (C)–(r); D-(p)
(2) (A)-(q); (B)-(s); (C)–(p); D-(r)
(3) (A)-(p); (B)-(q); (C)–(s); D-(r)
(4) (A)-(q); (B)-(s); (C)–(r); D-(p)

Answer (2)

Sol. (A) Fluorspar – CaF_2
(B) Cryolite – $Na_3[AlF_6]$
(C) Bauxite – $Al_2O_3.2H_2O$
(D) Dolomite – $MgCO_3.CaCO_3$

12. Which of the following element(s) is/are confirmed by appearance of blood red colour with $FeCl_3$ in Lassaigne's test?

(1) Presence of S only (2) Presence of N & S
(3) Presence of N only (4) Presence of P only

Answer (2)

Sol. $Na + C + N + S \rightarrow NaSCN$

$$Fe^{3+} + SCN^- \longrightarrow \underset{\text{Blood red}}{[Fe(SCN)]^{2+}}$$

13. Statement 1 : Electronegativity of group 14 elements decreases from Si to Pb.
Statement 2 : Group 14 has metals, metalloids and non-metals.

(1) Both Statements 1 and 2 are correct
(2) Both Statements 1 and 2 are incorrect
(3) Statement 1 is correct and Statement 2 is incorrect
(4) Statement 1 is incorrect and Statement 2 is correct

Answer (4)

Sol. Electronegativity generally decreases as we move down the group but Pb has higher electronegativity than Sn.

- C ⇒ non-metal
  Si and Ge ⇒ metalloids
  Sn and Pb ⇒ metals

E.N. of Sn = 1.8, Pb = 1.9

14. Hydrolysis of proteins gives which type of amino acid?

(1) α-Amino acid (2) β-Amino acid
(3) γ-Amino acid (4) δ-Amino acid

Answer (1)

Sol. Proteins on hydrolysis gives α-amino acid because α-amino acids are building block of proteins. It is also fact that amino acids contain both $-NH_2$ and $-COOH$ group.

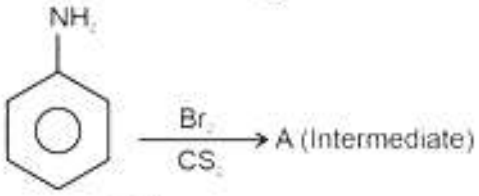
15. Statement 1 : Ionisation energy decreases in a period.
Statement 2 : In a period Z dominates over screening effect

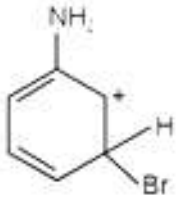
(1) Both statements 1 and 2 are correct
(2) Both statements 1 and 2 are incorrect
(3) Statement 1 is correct and statement 2 is incorrect
(4) Statement 1 is incorrect but statement 2 is correct

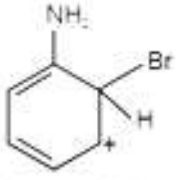
Answer (4)

Sol. Ionisation enthalpy increases in a period. Z dominates over screening effect (σ) in a period as $Z_{eff.}$ increases.

16. Consider the following reaction

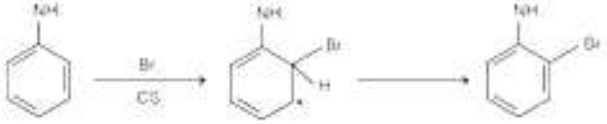


(1) 

(2) 

(3) Both (1) & (2)
(4) None of these

Answer (2)

Sol. 

17. Match the following

	Column I (Complexes)		Column II (Metals)
A.	Vitamin B_{12}	(p)	Ti
B.	Wilkinson catalyst	(q)	Co
C.	Ziegler-Natta catalyst	(r)	Fe
D.	Haemoglobin	(s)	Rh

(1) A(q), B(s), C(p), D(r) (2) A(s), B(q), C(r), D(p)
(3) A(q), B(p), C(r), D(s) (4) A(q), B(r), C(p), D(s)

Answer (1)

Sol. A. Vitamin B_{12} – Co
B. Wilkinson catalyst – $Rh([Rh(PPh_3)_3Cl])$
C. Ziegler-Natta catalyst – Ti $(TiCl_4 + Al(C_2H_5)_3)$
D. Haemoglobin – Fe

18. $K_2Cr_2O_7 + H_2O_2 + H_2SO_4 \xrightarrow{\text{ether cold conditions}}$ compound 'X'

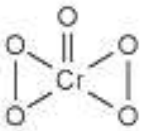
X is a chromium compound, what is the oxidation state of chromium in compound 'X'.

(1) +6 (2) +3
(3) +5 (4) +10

Answer (1)

Sol. $K_2Cr_2O_7 + H_2O_2 + H_2SO_4 \rightarrow \underset{(X)}{CrO_5} + K_2SO_4 + H_2O$

compound 'X' is $\Rightarrow CrO_5$



Oxidation state of chromium = +6.

19. $xCl_2 + yOH^- \longrightarrow zCl^- + pClO^-$

Balance the above reaction and find out values of x, y, z and p.

(1) x = 1, y = 2, z = 2, p = 1
(2) x = y = z = p = 1
(3) x = 1, y = 1, z = 2, p = 1
(4) x = 1, y = 2, z = 1, p = 1

Answer (4)

Sol. $\overset{0}{Cl_2} + \overset{0}{Cl_2} \longrightarrow \overset{-1}{2Cl^-} + \overset{+1}{2ClO^-}$

−2

+2

After balancing change in oxidation state,

$2Cl_2 \longrightarrow 2Cl^- + 2ClO^-$

Next, balance 'O' atoms,

$2Cl_2 + 4OH^- \longrightarrow 2Cl^- + 2ClO^- + 2H_2O$

Simplifying to get simplest ratios,

$Cl_2 + 2OH^- \longrightarrow Cl^- + ClO^- + H_2O$

x = 1, y = 2, z = 1, p = 1

20. For Rb(37) which of the following set of quantum numbers are correct for valence electron?

(1) $5, 0, 0, +\frac{1}{2}$ (2) $5, 0, 1, -\frac{1}{2}$
(3) $5, 0, 1, +\frac{1}{2}$ (4) $5, 1, 1, +\frac{1}{2}$

Answer (1)

Sol. $_{37}Rb = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 5s^1$

Last electron enters in 5s subshell

Value of quantum numbers

$n = 5, l = 0, m = 0, s = \pm\frac{1}{2}$

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Calculate the molarity of a solution having density = 1.5 g/mL %(w/w) of solute is 36% and molecular weight of solute is 36 g/mol.

Answer (15)

Sol. Assume mass of solution = 100 g

Mass of solute = 36 gm

Moles of solute = 1

$$\text{Molarity} = \frac{1\times1000}{\left(\frac{100}{1.5}\right)} = \frac{1000}{100}\times1.5 = 15$$

22. Given $K_{net} = \frac{K_1K_2}{K_3}$ when E_{a_1} = 40 kJ/mol

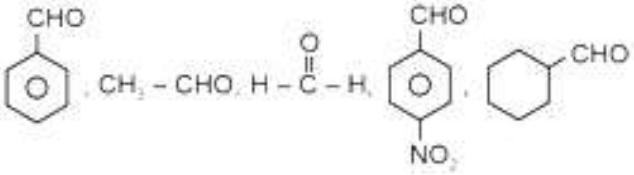
E_{a_2} = 50 kJ/mol, E_{a_3} = 60 kJ/mol.

Calculate value of $(E_a)_{net}$ in kJ/mol

Answer (30)

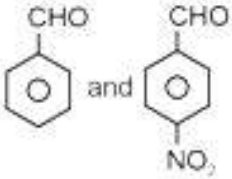
Sol. $(E_a)_{net} = E_{a_1} + E_{a_2} - E_{a_3}$
$= 40 + 50 - 60$
$= 90 - 60$
$= 30$ kJ/mol

23. Positive Fehling solution test is given by



Answer (3)

Sol. Fehling solution test can be given by aldehyde except aromatic aldehyde.



can't give Fehling solution test,

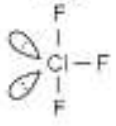
other all three given can give Fehling solution test.

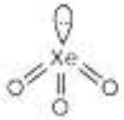
24. How many of the following compounds have one lone pair in central atom?

ClF_3, XeO_3, BrF_5, XeF_4, O_3, NH_3

Answer (4)

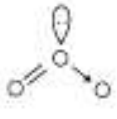
Sol.


lp = 2


lp = 1


lp = 1


lp = 2


lp = 1


lp = 1

25. How many of the following species have bond order = 1 and are paramagnetic as well?

He_2^{2+}; O_2^{2-}; Ne_2^{2+}; F_2; B_2; H_2; O_2^{2+}

Answer (1)

Sol. B_2 have bond order equal to 1 and also paramagnetic.

He_2^{2+}; O_2^{2-}; Ne_2^{2+}; F_2; H_2 have bond order equal to 1 but are diamagnetic.

O_2^{2+} have bond order equal to 3.

26. How many of the following compound contain sulphur atom?

Pyrrole, Furan, Thiophene, Cysteine, Tyrosine, Pyridine

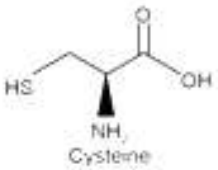
Answer (2)

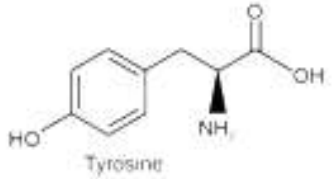
Sol.

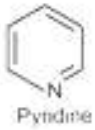

Pyrrole


Furan


Thiophene


Cysteine


Tyrosine


Pyridine

Thiophene and cysteine contain sulphur atom.

27. Through a $ZnSO_4$ solution, 0.015 A current was passed for 15 minutes. What is the mass of Zn deposited? (in mg)

(Atomic weight of Zn = 65.4)

Answer (5)

Sol. Charge passed = It

$= 0.015 \times 15 \times 60$ C

Moles of electrons passed = $\frac{0.015 \times 15 \times 60}{96500}$

Moles of Zn deposited = $\frac{1}{2} \times \frac{0.015 \times 15 \times 60}{96500}$

$= 0.00007$

Mass of Zn deposited = 0.00007×65.4 g = 4.58 mg

28. Osmotic pressure at 273 K is 7×10^5 Pa, then what will be the value of x, if its osmotic pressure at 283 K is $x \times 10^4$ Pa?

Answer (73)

Sol. $\pi_1 = iCRT_1$

$\pi_2 = iCRT_2$

$\frac{\pi_1}{T_1} = \frac{\pi_2}{T_2}$

$\pi_2 = \frac{\pi_1 \times T_2}{T_1}$

$= \frac{7 \times 10^5 \times 283}{273}$

$= 7.256 \times 10^5$ Pa

$= 72.56 \times 10^4$ Pa

$\pi_2 = x \times 10^4$

$\therefore \quad x = 72.56 \approx 73$

29. K_p for the given reaction is $(36 \times 10^{-2}\ atm^{-1})$. Find out K_c (M^{-1}) (nearest integer).

$(2NO_2 \rightleftharpoons N_2O_4)$

(R = 0.0821 atm.L/mol.K)

(T = 300 K)

Answer (9)

Sol. $K_p = K_c(RT)^{\Delta ng}$

$36 \times 10^{-2} = K_c(0.0821 \times 300)^{-1}$

$K_c = 0.36 \times 0.0821 \times 300 = 8.86 \approx 9$

30. ??

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let a die rolled till 2 is obtained. The probability that 2 obtained on even numbered toss is equal to

(1) $\frac{5}{11}$ (2) $\frac{5}{6}$

(3) $\frac{1}{11}$ (4) $\frac{6}{11}$

Answer (1)

Sol. $P(2 \text{ obtained on even numbered toss}) = k(\text{let})$

$$P(2) = \frac{1}{6}$$

$$P(\bar{2}) = \frac{5}{6}$$

$$k = \frac{5}{6}\times\frac{1}{6} + \left(\frac{5}{6}\right)^3 \times\frac{1}{6} + \left(\frac{5}{6}\right)^5 \times\frac{1}{6} + \ldots$$

$$= \frac{\frac{5}{6}\times\frac{1}{6}}{1-\left(\frac{5}{6}\right)^2}$$

$$= \frac{5}{11}$$

2. $\lim\limits_{x\to\frac{\pi}{2}^-} \frac{\int_{x^3}^{\left(\frac{\pi}{2}\right)^2} \cos t^{1/3}\,dt}{\left(x-\frac{\pi}{2}\right)^2}$

(1) $\frac{3\pi^2}{4}$ (2) $\frac{3\pi}{4}$

(3) $\frac{3\pi^2}{8}$ (4) $\frac{3\pi}{8}$

Answer (3)

Sol. $\lim\limits_{h\to 0} \frac{\int_{\left(\frac{\pi}{2}-h\right)^3}^{\left(\frac{\pi}{2}\right)^3} \cos\left(t^{1/3}\right)dt}{h^2}$

$$= \lim_{h\to 0} \frac{0 + 3\left(\frac{\pi}{2}-h\right)^2 \cos\left(\frac{\pi}{2}-h\right)}{2h}$$

$$= \lim_{h\to 0} \frac{3\left(\frac{\pi}{2}-h\right)^2 \sin h}{2h}$$

$$= \frac{3\pi^2}{8}$$

3. Consider the equation $4\sqrt{2}x^3 - 3\sqrt{2}x - 1 = 0$.

Statement 1: Solution of this equation is $\cos\frac{\pi}{12}$.

Statement 2: This equation has only one real solution.

(1) Both statement 1 and statement 2 are true

(2) Statement 1 is true but statement 2 is false

(3) Statement 1 is false but statement 2 is true

(4) Both statement 1 and statement 2 are false

Answer (2)

Sol. $12x = \pi$

$$\Rightarrow 3x = \frac{\pi}{4}$$

$$\cos 3x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 4\cos^3 x - 3\cos x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 4\sqrt{2}\cos^3 x - 3\sqrt{2}\cos x - 1 = 0$$

$x = \frac{\pi}{12}$ is the solution of above equation.

$\therefore$ Statement 1 is true

$f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$

$f'(x) = 12\sqrt{2}x^2 - 3\sqrt{2} = 0$

$$\Rightarrow x = \pm\frac{1}{2}$$

$$f\left(-\frac{1}{2}\right) = -\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}} - 1 = \sqrt{2} - 1 > 0$$

$f(0) = -1 < 0$

$\therefore$ one root lies in $\left(-\frac{1}{2}, 0\right)$, one root is $\cos\frac{\pi}{12}$ which is positive. As the coefficients are real, therefore all the roots must be real.

$\therefore$ Statement 2 is false.

4. If $|2A|^3 = 2^{21}$

and $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ then α is (if $\alpha, \beta \in I$)

(1) 5 (2) 3

(3) 9 (4) 17

Answer (1)

Sol. $|2A| = 2^7$

$8|A| = 2^7$

$|A| = 2^4$

Now $|A| = \alpha^2 - \beta^2 = 2^4$

$\alpha^2 = 16 + \beta^2$

$\alpha^2 - \beta^2 = 16$

$(\alpha - \beta)(\alpha + \beta) = 16$

$\Rightarrow \alpha + \beta = 8$ and

$\alpha - \beta = 2$

$\Rightarrow \alpha = 5$, and $\beta = 3$

5. In a 64 terms GP if sum of total terms is seven times sum of odd terms, then common ratio is

(1) 3 (2) 4

(3) 5 (4) 6

Answer (4)

Sol. $a, ar, ar^2, \ldots\ldots, ar^{63}$

$a + ar + ar^2 + \ldots\ldots + ar^{63} = 7\,[a + ar^2 + ar^4 + \ldots\ldots + ar^{62}]$

$$\frac{a(1-r^{64})}{(1-r)} = 7\frac{a(1-r^{64})}{(1-r^2)}$$

$1 + r = 7$

$r = 6$

6. If $\frac{dy}{dx} - \left(\frac{\sin 2x}{1+\cos^2 x}\right)y = \frac{\sin x}{1+\cos^2 x}$ and $y(0) = 0$ then $y\left(\frac{\pi}{2}\right)$ is

(1) -1 (2) 1

(3) 0 (4) 2

Answer (2)

Sol. $\frac{dy}{dx} - \left(\frac{\sin 2x}{1+\cos^2 x}\right)y = \frac{\sin x}{1+\cos^2 x}$

$IF = e^{-\int \frac{\sin 2x\,dx}{1+\cos^2 x}}$

$= e^{\ln(1+\cos^2 x)} = (1+\cos^2 x)$

So, $y(1 + \cos^2 x) = \int \frac{\sin x}{(1+\cos^2 x)} \cdot (1+\cos^2 x)dx$

$y(1 + \cos^2 x) = -\cos x + c$

$\because\ y(0) = 0$

$0 = -1 + c$

$\Rightarrow\ c = 1$

$$y = \frac{1-\cos x}{1+\cos^2 x}$$

Now, $y\left(\frac{\pi}{2}\right) = 1$

7. $4\cos\theta + 5\sin\theta = 1$

Then find $\tan\theta$, where $\theta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

(1) $\frac{10-\sqrt{10}}{6}$ (2) $\frac{10-\sqrt{10}}{12}$

(3) $\frac{\sqrt{10}-10}{6}$ (4) $\frac{\sqrt{10}-10}{12}$

Answer (4)

Sol. $16\cos^2\theta + 25\sin^2\theta + 40\sin\theta\cos\theta = 1$

$16 + 9\sin^2\theta + 20\sin 2\theta = 1$

$$16 + 9\left(\frac{1-\cos 2\theta}{2}\right) + 20\sin 2\theta = 1$$

$$\frac{-9}{2}\cos 2\theta + 20\sin 2\theta = \frac{-39}{2}$$

$-9\cos 2\theta + 40\sin 2\theta = -39$

$$-9\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + 40\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = -39$$

$48\tan^2\theta + 80\tan\theta + 30 = 0$

$24\tan^2\theta + 40\tan\theta + 15 = 0$

$$\tan\theta = \frac{-40 \pm \sqrt{(40)^2 - 15\times 24\times 4}}{2\times 24}$$

$$\tan\theta = \frac{-40 \pm \sqrt{160}}{2\times 24}$$

$$= \frac{-10 \pm \sqrt{10}}{12}$$

$$\Rightarrow \tan\theta = \frac{\sqrt{10}-10}{12}, \quad \tan\theta = \frac{-\sqrt{10}-10}{12}$$

So $\tan\theta = -\frac{\sqrt{10}-10}{12}$ will be rejected as $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Option (4) is correct.

8. In an increasing arithmetic progression $a_1, a_2, \ldots a_6$ if $a_6 = 2$ and product of a_1, a_5 and a_4 is greatest, then the value of d is equal to

(1) 1.6 (2) 1.8

(3) 0.6 (4) 2.0

Answer (1)

Sol. First term = a

Common difference = d

Given: $a + 5d = 2$ ... (1)

Product $(P) = (a_1a_5a_4) = a(a + 4d)(a + 3d)$

Using (1)

$P = (2 - 5d)(2 - d)(2 - 2d)$

$$\Rightarrow \frac{dP}{dd} = (2-5d)(2-d)(-2) + (2-5d)(2-2d)(-1) + (-5)(2-d)(2-2d)$$

$= -2[(d - 2)(5d - 2) + (d - 1)(5d - 2) + 5(d - 1)(d - 2)]$

$= -2[5d^2 + 4 - 12d + 5d^2 + 2 - 7d + 5d^2 + 10 - 15d]$

$= -2[15d^2 - 34d + 16]$

$\Rightarrow d = \frac{8}{5}$ or $\frac{2}{3}$

at $\left(\frac{8}{5}\right)$, product attains maxima

$\Rightarrow \boxed{d = 1.6}$

9. If relation R : (a, b) $R(c, d)$ is only if $ad - bc$ is divisible by 5 $(a, b, c, d \in Z)$ then R is

(1) Reflexive

(2) Symmetric, Reflexive but not Transitive

(3) Reflexive, Transitive but not symmetric

(4) Equivalence relation

Answer (2)

Sol. Reflexive : for (a, b) R (a, b)

$\Rightarrow$ $ab - ab = 0$ is divisible by 5.

So (a, b) $R(a, b)$ $\forall$ $a, b \in Z$

$\therefore$ R is reflexive

Symmetric :

For (a, b) $R(c, d)$

If $ad - bc$ is divisible by 5.

Then $bc - ad$ is also divisible by 5.

$\Rightarrow$ (c, d) $R(a, b)$ $\forall$ $a, b, c, d \in Z$

$\therefore$ R is symmetric

Transitive :

If (a, b) $R(c, d) \Rightarrow ad - bc$ divisible by 5

and (c, d) R $(e, f) \Rightarrow cf - de$ divisible by 5

$ad - bc = 5k_1$ k_1 and k_2 are integers

$cf - de = 5k_2$

$afd - bcf = 5k_1f$

$bcf - bde = 5k_2b$

$afd - bde = 5(k_1f + k_2b)$

$d(af - be) = 5(k_1f + k_2b)$

$\Rightarrow$ $af - be$ is not divisible by 5 for every $a, b, c, d, e, f \in Z$.

$\therefore$ R is not transitive

For e.g., take $a = 1, b = 2, c = 5, d = 5, e = 2, f = 2$

10. Let $f(x) = \begin{cases} 2 + 2x, & x \in (-1, 0) \\ 1 - \frac{x}{3}, & x \in [0, 3) \end{cases}$

$$g(x) = \begin{cases} x, & x \in [0, 1) \\ -x, & x \in (-3, 0) \end{cases}$$

The range of $fog(x)$ is

(1) [0, 1] (2) [–1, 1]

(3) (0, 1] (4) (–1, 1)

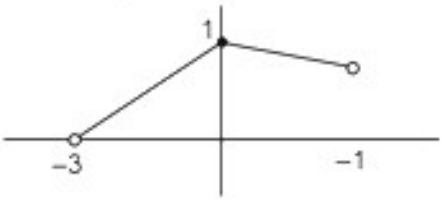
Answer (3)

Sol. $f(x) = \begin{cases} 2 + 2x, & x \in (-1, 0) \\ 1 - \frac{x}{3}, & x \in [0, 3) \end{cases}$

$$g(x) = \begin{cases} x, & x \in [0, 1) \\ -x, & x \in (-3, 0) \end{cases} \Rightarrow g(x) = |x|,\ x \in (-3, 1)$$

$$f(g(x)) = \begin{cases} 2 + 2|x|, & |x| \in (-1, 0) \Rightarrow x \in \phi \\ 1 - \frac{|x|}{3}, & |x| \in [0, 3) \Rightarrow x \in (-3, 1) \end{cases}$$

$$f(g(x)) = \begin{cases} 1 - \frac{x}{3}, & x \in [0, 1) \\ 1 + \frac{x}{3}, & x \in (-3, 0) \end{cases}$$



Range of $fog(x)$ is [0, 1]

11. If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1 + \pi^x} + \frac{1 + \sin^2 x}{1 + e^{(\alpha - \beta)x}} \right) dx = \frac{\pi}{4}(\pi + \alpha) - 2$

Then the value of 'α' is equal to

(1) 1 (2) 2

(3) 3 (4) 4

Answer (3)

Sol. Given

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{(\sin x)^{2023}}}\right)dx=\frac{\pi}{4}(\pi+\alpha)-2$$

$$\int_{0}^{\frac{\pi}{2}}\left\{\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{(\sin x)^{2023}}}\right)+\left(\frac{x^2\cos x}{1+\pi^{-x}}+\frac{1+\sin^2 x}{1+e^{-(\sin x)^{2023}}}\right)\right\}dx$$

$$=\frac{\pi}{4}(\pi+\alpha)-2$$

$$\int_{0}^{\frac{\pi}{2}}\left(x^2\cos x+1+\sin^2 x\right)dx=\frac{\pi}{4}(\pi+\alpha)-2$$

$$\int_{0}^{\frac{\pi}{2}}x^2\cos x\,dx+\int_{0}^{\frac{\pi}{2}}(1+\sin^2 x)dx=\frac{\pi}{4}(\pi+\alpha)-2 \quad ...(1)$$

Let $I_1=\int_{0}^{\frac{\pi}{2}}(1+\sin^2 x)dx$

$$I_1=\int_{0}^{\frac{\pi}{2}}1\cdot dx+\int_{0}^{\frac{\pi}{2}}\left(\frac{1-\cos 2x}{2}\right)dx$$

$$I_1=\frac{\pi}{2}+\frac{1}{2}\left[\frac{\pi}{2}+0\right]$$

$$I_1=\frac{\pi}{2}+\frac{\pi}{4}$$

$$\boxed{I_1=\frac{3\pi}{4}}$$

Let $I_2=\int_{0}^{\frac{\pi}{2}}x^2\cos x\,dx$

$$I_2=\left[x^2(\sin x)-\int 2x\int\cos x\,dx\right]_0^{\frac{\pi}{2}}$$

$$I_2=\left[x^2(\sin x)-2\int x\sin x\right]_0^{\frac{\pi}{2}}$$

$$I_2=\left[x^2\sin x-2\left(x(-\cos x)+\int\cos x\right)\right]_0^{\frac{\pi}{2}}$$

$$I_2=\left[x^2\sin x-2(-x\cos x+\sin x)\right]_0^{\frac{\pi}{2}}$$

$$I_2=\left(\frac{\pi^2}{4}-2\right)$$

$\therefore$ Put I_1 and I_2 in (1)

$$\therefore \quad \frac{\pi^2}{4}-2+\frac{3\pi}{4}$$

$$\frac{\pi^2}{4}+\frac{3\pi}{4}-2$$

$$\frac{\pi}{4}(\pi+3)-2$$

$$\therefore \quad \boxed{\alpha=3}$$

12. Area under the curve $x^2+y^2=169$ and below the line $5x-y=13$ is

(1) $\frac{169\pi}{4}-\frac{65}{2}+\frac{169}{2}\sin^{-1}\frac{12}{13}$

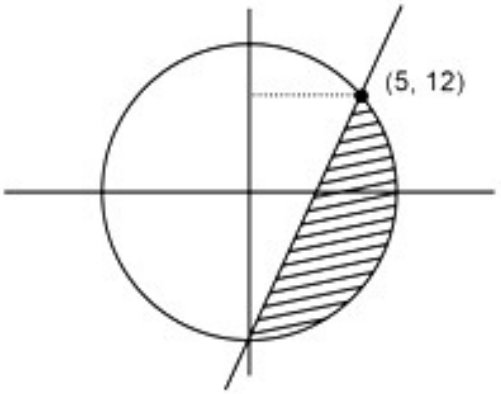
(2) $\frac{169\pi}{4}+\frac{65}{2}-\frac{169}{2}\sin^{-1}\frac{12}{13}$

(3) $\frac{169}{4}-\frac{65}{2}+\frac{169}{2}\sin^{-1}\frac{13}{14}$

(4) $\frac{169\pi}{4}+\frac{65}{2}+\frac{169}{2}\sin^{-1}\frac{13}{14}$

Answer (1)

Sol.



$$\text{Area}=\frac{\pi(13)^2}{2}-\left[\frac{1}{2}\times 25\times 5+\int_{12}^{13}\sqrt{\left(169-y^2\right)}\cdot dy\right]$$

$$=\frac{169\pi}{2}-\left[\frac{125}{2}+\left[\frac{y}{2}\sqrt{169-y^2}+\frac{169}{2}\sin^{-1}\frac{y}{13}\right]_{12}^{13}\right]$$

$$=\frac{169}{2}\pi-\frac{125}{2}-\left[\frac{169}{2}\times\frac{\pi}{2}-6\times 5-\frac{169}{2}\sin^{-1}\frac{12}{13}\right]$$

$$=\frac{169\pi}{4}-\frac{65}{2}+\frac{169}{2}\sin^{-1}\frac{12}{13}$$

13. If $f(x)=\frac{(2^x+2^{-x})(\tan x)\sqrt{\tan^{-1}(2x^2-3x+1)}}{(7x^2-3x+1)^3}$, then $f'(0)$ is equal to

(1) $\sqrt{\pi}$ (2) $\sqrt{\frac{\pi}{4}}$

(3) π (4) $2\cdot\pi^{3/2}$

Answer (1)

Sol. $f(x)=\dfrac{(2^x+2^{-x})\tan x\sqrt{\tan^{-1}(2x^2-3x+1)}}{(7x^2-3x+1)^3}$

$f(x)=(2^x+2^{-x}).\tan x.\sqrt{\tan^{-1}(2x^2-3x+1)}.(7x^2-3x+1)^{-3}$

$f'(x)=(2^x+2^{-x})\sec^2 x\sqrt{\tan^{-1}(2x^2-3x+1)}(7x^2-3x+1)^{-3}+\tan x\,(Q(x))$

$\therefore f'(0)=2.1.\sqrt{\dfrac{\pi}{4}}.1$

$=\sqrt{\pi}$

14. $\int\dfrac{(\sin x-\cos x)\sin^2 x}{\sin x\cos^2 x+\tan x\sin^3 x}dx$ is equal to

(1) $\dfrac{\ln|\sin^3 x-\cos^3 x|}{3}+c$

(2) $\dfrac{\ln|\sin^3 x+\cos^3 x|}{3}+c$

(3) $\dfrac{\ln|\sin^3 x-\cos^3 x|}{2}+c$

(4) $\dfrac{\ln|\sin^3 x+\cos^3 x|}{4}+c$

Answer (2)

Sol. $\int\dfrac{(\sin x-\cos x)\sin^2 x}{\tan x(\sin^3 x+\cos^3 x)}dx$

$\int\dfrac{(\sin x-\cos x)\sin x\cos x}{\sin^3 x+\cos^3 x}dx$, put $\sin^3x+\cos^3x=t$

$(3\sin^2x\cdot\cos x-3\cos^2x\sin x)\,dx=dt$

$\Rightarrow\ \dfrac{1}{3}\int\dfrac{dt}{t}$

$=\dfrac{\ln t}{3}+c$

$=\dfrac{\ln|\sin^3 x+\cos^3 x|}{3}+c$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. $\dfrac{{}^{11}C_1}{2}+\dfrac{{}^{11}C_2}{3}+.....+\dfrac{{}^{11}C_9}{10}=\dfrac{m}{n}$

Then $m+n$ is

Answer (2041)

Sol. $(1+x)^{11}={}^{11}C_0+{}^{11}C_1x+{}^{11}C_2x^2+....+{}^{11}C_{11}x^{11}$

$\int_0^1(1+x)^{11}dx={}^{11}C_0x+\dfrac{{}^{11}C_1x^2}{2}+\dfrac{{}^{11}C_2x^3}{3}+....$

$+\dfrac{{}^{11}C_9x^{10}}{10}+\dfrac{{}^{11}C_{10}x^{11}}{11}+\dfrac{{}^{11}C_{11}x^{12}}{12}\Big]_0^1$

$\dfrac{(1+x)^{12}}{12}\Big]_0^1={}^{11}C_0+\dfrac{{}^{11}C_1}{2}+\dfrac{{}^{11}C_2}{3}+....+\dfrac{{}^{11}C_9}{10}+\dfrac{{}^{11}C_{10}}{11}+\dfrac{{}^{11}C_{11}}{12}$

$\dfrac{2^{12}-1}{12}-1-1-\dfrac{1}{12}=\dfrac{{}^{11}C_1}{2}+\dfrac{{}^{11}C_2}{3}+....+\dfrac{{}^{11}C_{10}}{11}$

$=\dfrac{2^{12}-2-24}{12}$

$=\dfrac{2^{12}-26}{12}=\dfrac{4070}{12}=\dfrac{2035}{6}=\dfrac{m}{n}$

$m+n=2035+6=2041$

22. Rank of the word 'GTWENTY' in dictionary is

Answer (553)

Sol. Start with

(1) $\bar{E}:\dfrac{6!}{2!}=360$

(2) $\overline{GE}:\dfrac{5!}{2!}$, $\overline{GN}:\dfrac{5!}{2!}$

(3) GTE : 4!, GTN: 4!, GTT : 4!

(4) GTWENTY = 1

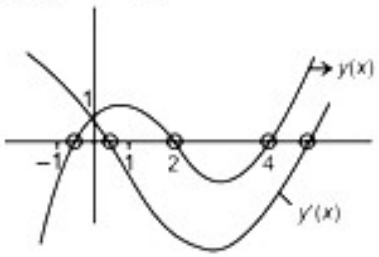
$\Rightarrow$ 360 + 60 + 60 + 24 + 24 + 24 + 1 = 553

23. Curve $y=2^x-x^2$, $y(x)$ & $y'(x)$ cut x-axis in M & N number of points respectively, find $M+N$.

Answer (5)

Sol. $y(x)=2^x-x^2$

$y'(x)=2^x\log 2-2x$



$M=3$

$N=2$

$M+N=5$

24. Given data

60, 60, 44, 58, 68, α, β, 56 has mean 58, variance = 66.2 then find $\alpha^2 + \beta^2$

Answer (7182)

Sol. Variance $= \frac{\Sigma x^2}{n} - (\bar{x})^2$

$$\frac{60^2 + 60^2 + 44^2 + 58^2 + 68^2 + \alpha^2 + \beta^2 + 56^2}{8} - (58)^2 = 66.2$$

$$\frac{7200 + 1936 + 3364 + 4624 + 3136 + \alpha^2 + \beta^2}{8} - 3364 = 66.2$$

$$2532.5 + \frac{\alpha^2 + \beta^2}{8} - 3364 = 66.2$$

$\alpha^2 + \beta^2 = 897.7 \times 8$

$= 7181.6$

25. If $|z + 1| = \alpha z + \beta (i + 1)$ and $z = \frac{1}{2} - 2i$, find $\alpha + \beta$.

Answer (3)

Sol. $\left|\frac{1}{2} - 2i + 1\right| = \alpha\left(\frac{1}{2} - 2i\right) + \beta(1 + i)$

$$\sqrt{\frac{9}{4} + 4} = \alpha\left(\frac{1}{2} - 2i\right) + \beta(1 + i)$$

$$\frac{5}{2} = \alpha\left(\frac{1}{2}\right) + \beta + i(-2\alpha + \beta)$$

$\frac{\alpha}{2} + \beta = \frac{5}{2}$ ...(1)

$-2\alpha + \beta = 0$ ...(2)

Solving (1) and (2)

$\frac{\alpha}{2} + 2\alpha = \frac{5}{2}$

$\frac{5}{2}\alpha = \frac{5}{2}$

$\alpha = 1$

$\beta = 2$

$\Rightarrow \alpha + \beta = 3$

26. If $\vec{a}, \vec{b}, \vec{c}$ are non-zero and $\vec{b}$ and $\vec{c}$ are non-collinear, $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 6\vec{c}$ is collinear with $\vec{a}$. If $\vec{a} + \alpha\vec{b} + \beta\vec{c} = 0$, then find $\alpha + \beta$.

Answer (35)

Sol. $\because \vec{a} + 5\vec{b}$ is collinear with $\vec{c}$

$\Rightarrow \vec{a} + 5\vec{b} = \lambda\vec{c}$ ...(1)

$\vec{b} + 6\vec{c}$ is collinear with $\vec{a}$

$\Rightarrow \vec{b} + 6\vec{c} = \mu\vec{a}$ ...(2)

From (1) and (2)

$\vec{b} + 6\vec{c} = \mu(\lambda\vec{c} - 5\vec{b})$

$\Rightarrow (1 + 5\mu)\vec{b} + (6 - \lambda\mu)\vec{c} = 0$

$\because \vec{b}$ and $\vec{c}$ are non-collinear

$\Rightarrow 1 + 5\mu = 0 \Rightarrow \mu = \frac{-1}{5}$ and

$6 - \lambda\mu = 0 \Rightarrow \lambda\mu = 6$

$\Rightarrow \lambda = -30$

Now,

$\vec{b} + 6\vec{c} = \frac{-1}{5}\vec{a}$

$5\vec{b} + 30\vec{c} = -\vec{a}$

$\left.\begin{aligned}\vec{a} + 5\vec{b} + 30\vec{c} &= 0\\ \vec{a} + \alpha\vec{b} + \beta\vec{c} &= 0\end{aligned}\right]$

On comparing

$\alpha = 5, \beta = 30 \Rightarrow \alpha + \beta = 35$

27.

28.

29.

30.