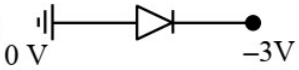
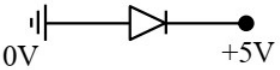
JEE (MAIN) 2024 DATE-27/01/2024 (SHIFT-1)

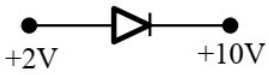
PHYSICS

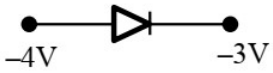
SECTION-A

1. Which among the following is forward biased:

(1) 

(2) 

(3) 

(4) 

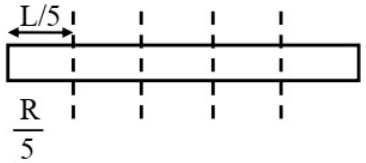
Ans. **(1)**

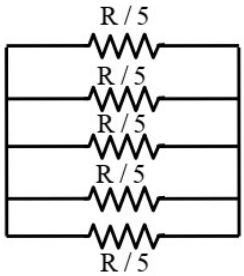
Sol. **Basic theory.**

2. A uniform and homogeneous rod has resistance R. If rod is cut into 5 equal parts and connected in parallel find equivalent resistance ?

Ans. $\frac{R}{25}$

Sol.





$\Rightarrow \quad \frac{R}{25}$ Answer

3. Acceleration due to earth on the surface is g_0. If mass of earth remains same but radius is half, then find the acceleration on the surface for new system :

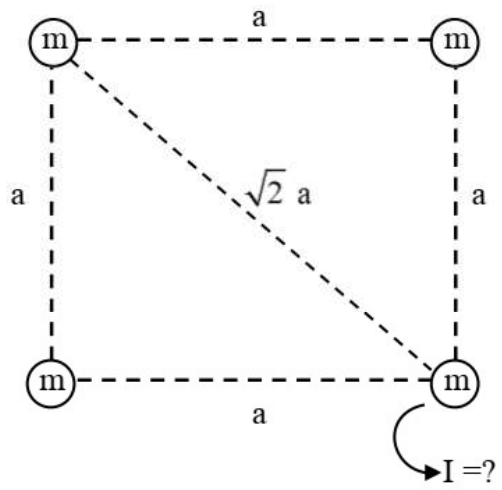
(1) $\frac{g_0}{2}$ (2) g_0 (3) $2\,g_0$ (4) $4\,g_0$

Ans. **(D)**

Sol. $g_0 = \frac{Gm}{R^2}$

$$g = \frac{Gm}{(R/2)^2} = \frac{4Gm}{R^2} = 4g_0$$

4. Find moment of inertia about an axis passing though one corner and perpendicular to the plane.



Ans. $4\,ma^2$

Sol. $I = ma^2 + ma^2 + m\left(\sqrt{2}\,a\right)^2 + 0 = 4\,ma^2$

5. Two particles having mass 4g & 25g have same kinetic energy. Find ratio of their momentum?

(1) $\frac{2}{5}$ (2) $\frac{2}{3}$ (3) $\frac{4}{5}$ (4) $\frac{3}{4}$

Ans. **(1)**

Sol. $\mathbf{KE_1 = KE_2}$

$$\frac{P_1^2}{2m_1} = \frac{P_2^2}{2m_2}$$

$$\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{4}{25}} = \frac{2}{5}$$

6. An object of mass 1000 kg is moving with 6 m/s. Find speed of object is mass 200 kg is added to it ?

(1) 4 m/s (2) 5 m/s (3) 8 m/s (4) 6 m/s

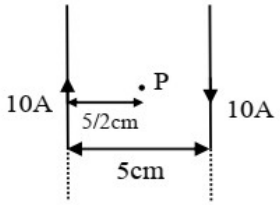
Ans. **(2)**

Sol. Linear momentum is conserved.

$1000 \times 6 = 1200\ (v_f)$

$\therefore \quad v_f = 5$ m/s

7. Two very long wire having current as shown. Find the magnetic field at point 'P' (in micro tesla).



Ans. **160**

Sol. $\mathbf{B} = \dfrac{\mu_0 I}{2\pi D} \times 2$

$\mathbf{B} = \dfrac{2\times10^{-7}\times10}{\frac{5}{2}\times10^{-2}} \times 2$

$B = 16 \times 10^{-5}$ Tesla

$B = 160\ \mu T$

8. If the electron revolving in the third Bohr's orbit of hydrogen species has radius R, then what will be its radius in fourth orbit in terms of R.

(1) $\dfrac{25R}{9}$ (2) $\dfrac{16R}{9}$ (3) $\dfrac{36R}{9}$ (4) $\dfrac{9R}{16}$

Ans. **(B)**

Sol. $\mathbf{R} = \dfrac{kn^2}{Z}$

$$\frac{R}{R'} = \frac{\frac{k3^2}{Z}}{\frac{k4^2}{Z}}$$

$\Rightarrow \quad \dfrac{R}{R'} = \dfrac{9}{16}$

$\Rightarrow \quad R' = \dfrac{16}{9}R$

9. A charge of magnitude $10^{-6}\mu C$ is placed at origin in x-y co-ordinate system. Find the potential difference between the two point $\left(\sqrt{3}, \sqrt{3}\right)$ and $\left(\sqrt{6}, 0\right)$. (Axis are in meters)

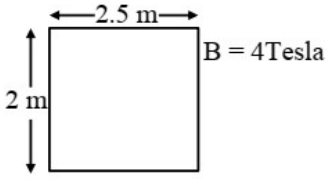
(1) $3\sqrt{3}\times10^{3}V$ (2) $\frac{3}{\sqrt{3}}\times10^{3}V$

(3) 0V (4) $2\sqrt{3}\times10^{3}V$

Ans. **(3)**

Sol. Same radial distance from origin Hence Potential is same at the two given point. Thus potential difference is zero

10. Magnetic field having magnitude 4 Tesla makes an angle 60° with perpendicular to loop and loop has been removed from magnetic field region within 10 seconds. Find average induced emf in loop in 10 seconds in Volts?

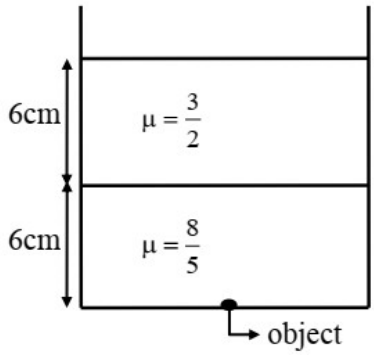


Ans. **1**

Sol. $e_{avg} = \frac{\Delta\phi}{\Delta t} = \frac{BA\cos\theta}{10}$

$= 4\times2\times\frac{5}{2}\times\frac{\cos60}{10} = 1$ volt

11. Find apparent depth of the object shown in figure ?



Ans. $\frac{31}{4}$

Sol. Apparent depth $= \frac{6}{3/2}+\frac{6}{8/5} = 4+\frac{15}{4} = \frac{31}{5}$cm

12. An EM wave is given by

$E = 200 \sin [1.5 \times 10^7 t - 0.05 x]$ N/C

Find the intensity of wave. [$\varepsilon_0 = 8.85 \times 10^{-12}$ SI units]

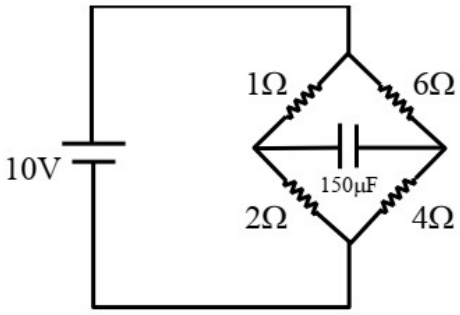
Ans. **53.1**

Sol. $\mathbf{I} = \frac{1}{2}\varepsilon_0 E_0^2 . C_{mid}$

$\mathbf{I} = \frac{1}{2} \times 8.85 \times 10^{-12} \times [200]^2 \frac{1.5 \times 10^7}{0.05}$

$\mathbf{I = 53.1\ W/m^2}$

13.

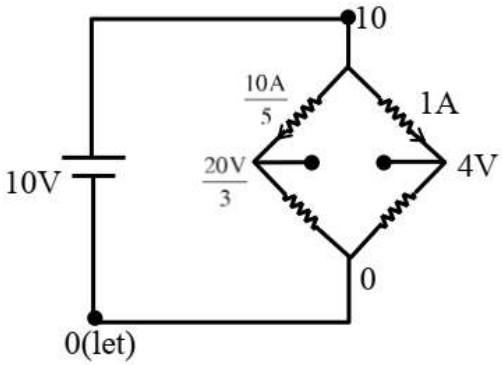


Find charge on capacitor at steady state?

(1) 200 μC (2) 300 μC (3) 400 μC (4) 500 μC

Ans. **(3)**

Sol.



$\therefore \Delta V)_{capacitor} = \left|4 - \frac{20}{3}\right| = \frac{8}{3} V$

$\therefore q = \frac{8}{3} \times 150 = \boxed{400 \mu C}$

14. A particle performs SHM with an amplitude 4 cm. Speed of particle at mean position is 10 cm/sec. Find position from mean where speed is 5 cm/sec

(1) 2 cm (2) $2\sqrt{3}$ cm (3) 0.5 cm (4) $\sqrt{3}$ cm

Ans. **(2)**

Sol. $10 \text{ cm/s} = A\omega$ ...(i)

$5 \text{ cm/s} = \omega\sqrt{A^2 - x^2}$ ...(ii) using (i) and (ii)

$\mathbf{x} = \frac{\sqrt{3}A}{2} = 2\sqrt{3}$ cm

15. Given :

$m = 0.08$ kg

$s_v = 0.17$ kcal/kg-°C

$\Delta T = 5°C$

Find change in internal energy (in Joule) of gas.

Ans. **284**

Sol. $\Delta U = ms_v\Delta T$

$\Delta U = 0.08 \times 0.17 \times 10^3 \times 5$

$\Delta U = 68$ cal

$\Delta U = 284.24$ Joule

16. A gas undergoes isothermal expansion from 30 dm^3 to 45 dm^3. Find heat absorbed by gas if external pressure is 10 kPa?

(1) 100 J (2) 150 J (3) 120 J (4) 200 J

Ans. **(C)**

Sol. $\Delta V = 0$

$\therefore \Delta Q = w$

$= nRT\ell n\left(\frac{V_2}{V_1}\right)$

$= P_1V_1\,\ell n\left(\frac{V_2}{V_1}\right)$

$= 10 \times 10^3 \times 30 \times 10^{-3}\,\ell n\left(\frac{3}{2}\right)$

$= 300 \times 0.4$

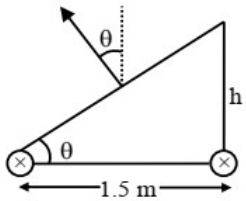
$= 120$ J

17. A banked road of radius 400 m is there with base separation between the rails is 1.5 m, if speed of a car for safe turning is 12 m/s, then find height of one rail w.r.t to second rail?

(1) h = 0.054 m (2) h = 0.1 m (3) h = 0.001 m (4) h = 0.2 m

Ans. **(1)**

Sol.



$N\cos\theta = mg$

$N\sin\theta = \frac{mv^2}{r}$

$\tan\theta = \frac{v^2}{rg}$

$\frac{h}{1.5} = \frac{12\times12}{400\times10}$

$h = \frac{12\times12}{4000}\times\frac{3}{2} = \frac{54}{1000}$

$h = 0.054$ m

18. A particle is moving from origin with initial velocity $5\hat{i}$ m/s and constant acceleration $3\hat{i}+2\hat{j}$ m/s^2 When position of particle is 84 m, its velocity is $\sqrt{\alpha}$ m/s. Find out α :

Ans. **673**

Sol. $x = u_x t + \frac{1}{2}a_x t^2$

$84 = 5t + \frac{3}{2}t^2$

$t = 6$ sec.

$\vec{v} = \vec{u} + \vec{a}t$

$\vec{v} = 5\hat{i} + (3\hat{i}+2\hat{j})\ 6$

$= 23\hat{i} + 12\hat{j}$

$= 529 + 144$

$= \sqrt{673}$ m/s

$\alpha = 673$

19. **Statement-1 :** Angular momentum and Plank constant have same dimensions.

Statement-2 : Moment of force and linear momentum have same dimensions.

(1) Both statements are true

(2) Both statements are false

(3) Statement 1 is true and 2nd is false

(4) Statement 2 is true and 1st is false

Ans. **(3)**

Sol. $\mathbf{L} = \frac{nh}{2\pi}, \qquad F = \frac{dp}{dt}$

$[L] = M^1L^2T^{-1}$

$[h] = ML^2T^{-1}$

$[\tau] = M^1L^2T^{-2}$

$[P] = M^1L^1T^{-1}$

20. A proton is moving in gravity free space with constant velocity v and goes undeviated. What can be the possible conditions.

(A) $E = 0, B = 0$

(B) $E = 0, B \neq 0$

(C) $E \neq 0, B = 0$

(D) $E \neq 0, B \neq 0$

(1) A, B, D (2) A, B, C (3) A, B, C, D (4) B, C, D

Ans. **(1)**

21. $\mathbf{S_1} \rightarrow$ Viscosity coefficient of gas is less than liquid.

$\mathbf{S_2} \rightarrow$ Surface tension decreases if insoluble impurities are added.

(1) S_1 is true, S_2 is true

(2) S_1 is false, S_2 is false

(3) S_1 is true, S_2 is false

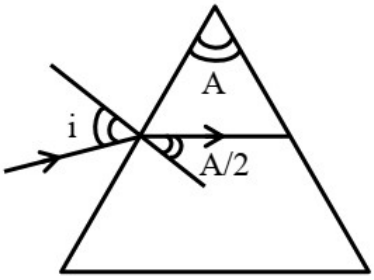
(4) S_1 is false, S_2 is true

Ans. **(1)**

22. There in a prism of apex angle of 'A'. Its refractive index is equal to Cot $\frac{A}{2}$, then find minimum angle of deviation?

Ans. **2**

Sol.



$1 \sin i = \mu \sin \frac{A}{2}$

$\sin i = \left(\cot \frac{A}{2}\right) \sin \frac{A}{2}$

$\sin i = \cos \frac{A}{2} = \sin\left(\frac{\pi}{2} - \frac{A}{2}\right)$

$i = \frac{\pi}{2} - \frac{A}{2}$

$\delta_{min} = 2i - A = \pi - 2A$

Alternate Solution

$n = \frac{\sin \frac{A + \delta_{min}}{2}}{\sin \frac{A}{2}}$

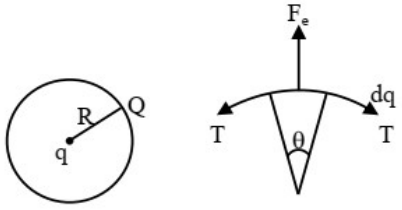
$\frac{\cos \frac{A}{2}}{\sin \frac{A}{2}} = \frac{\sin \frac{A + \delta_{min}}{2}}{\sin \frac{A}{2}}$

$\Rightarrow \delta_{min} = \pi - 2A$

23. A point charge q is placed at a centre of a charged ring of total charge Q. Find tension in the ring.

Ans. $\frac{KQq}{2\pi R^2}$

Sol.



$$\frac{kqdq}{R^2} = 2T\sin\left(\frac{\theta}{2}\right) \qquad \theta \simeq \text{small}$$

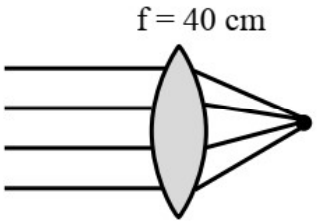
$$\frac{kqQ\theta}{2\pi R^2} = T\theta \qquad \text{Also } \frac{Q}{dq} = \frac{2\pi}{\theta}$$

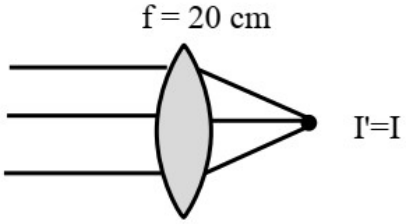
$$T = \frac{KQq}{2\pi R^2}$$

24. Light in incident on a convex lens of focal length 40 cm. And a metal plate is placed on focus of lens & photo current is measure to be I. Find new photocurrent if lens is replaced by another lens focal length of 20 cm & metal plate is kept on its focus?

Ans. I'= I

Sol.

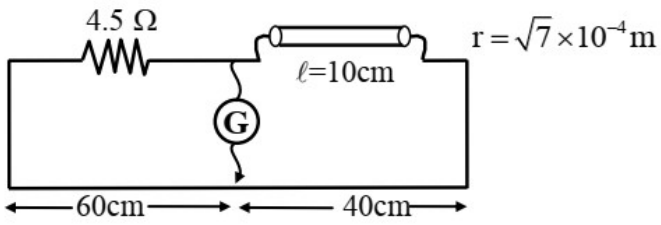




25. In meter bridge experiment there is a resistance in right slot of length 10 cm and radius of cross section is $\sqrt{7} \times 10^{-4}$ m. In left slot these is a resistance of 4.5 Ω. If balance length from left is 60 cm. If unknown resistivity is $x \times 10^{-7}$. Find 'x'.

Ans. **66**

Sol.



$$\frac{60}{40}=\frac{4.5}{R} \quad \Rightarrow \quad R = 3\Omega$$

$$R = \frac{\rho \ell}{A}$$

$$3 = \rho \times \frac{1}{10 \times \pi \times 7 \times 10^{-8}} \Rightarrow \quad \rho = 21\pi \times 10^{-7} = 21 \times \frac{20}{7} \times 10^{-7} = 66 \times 10^{-7} = x \times 10^{-7}$$

$$x = 66$$

26. Spherometer can't be used for measurement of :

(1) Radius of curvature of convex mirror

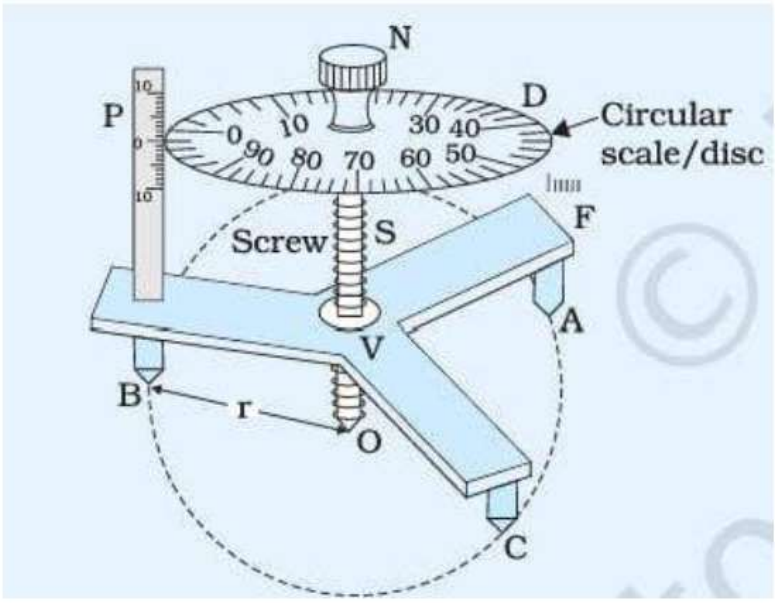
(2) Radius of curvature of concave mirror

(3) Thickness of capacitor plates

(4) Specific rotation of liquid

Ans. (4)

Sol. Spherometer is used to measure radius of curvature of any spherical surface and any small thickness.



CHEMISTRY

1. Which of the following has maximum magnetic moment?

(1) $3d^3$ (2) $3d^6$ (3) $3d^7$

Ans. **(2)**

2. Mass of methane required to produce 22 g CO_2 upon combustion is _____.

Ans. **(8)**

Sol. Moles of $CO_2 = \frac{22}{44} = 0.5 \therefore n_{CH_4} = 0.5 \quad \therefore m_{CH_4} = 8g$

3. Assertion : Boron has very high melting point (2453 K)

Reason : Boron has strong crystalline lattice.

Ans. A-T ; R-T ;

Exp. $\rightarrow$ Right

4. Sum of bond order of CO & NO^+ is :

Ans. **(6)**

Sol. CO : 3 ; NO^+ : 3

5. How many of following have +4 oxidation number of central atom:

$BaSO_4$, $SOCl_2$, SF_4, H_2SO_3, $H_2S_2O_7$, SO_3

Ans. **(3)**

Sol. $SOCl_2$, SF_4, H_2SO_3

6. $PbCrO_4$ + NaOH (hot excess) $\longrightarrow$?

Product is :

(1) dianionic ; CN = 4 (2) tetra-anionic ; CN = 6

(3) dianionic ; CN = 6 (4) tetra-anionic ; CN = 4

Ans. **(4)**

7. For negative deviation from Raoult's law :

(1) BP increases ; VP increases (2) BP decreases ; VP increases

(3) BP decreases ; VP decreases (4) BP increases ; VP decreases

Ans. **(4)**

8. $NaCl + H_2SO_4 + K_2Cr_2O_7 \longrightarrow$ Products

Above reaction gives red fumes (A) which on hydrolysis with aqueous NaOH gives yello solution (B). Compounds (A) and (B) are :

Ans. CrO_2Cl_2, Na_2CrO_4

Sol. $NaCl + H_2SO_4 + K_2Cr_2O_7 \rightarrow \underset{(A)}{CrO_2Cl_2} + Na_2SO_4 + K_2SO_4 + H_2O$

$CrO_2Cl_2 + NaOH\ (aq.) \rightarrow Na_2CrO_4 + NaCl + H_2O$

(B)

9. Order of spin only magnetic moment for

$[FeF_6]^{-3}$ $[V(H_2O)_6]^{+2}$ $[Fe(H_2O)_6]^{+2}$

(P) (Q) (R)

(1) P > R > Q (2) P > Q > R (3) R > Q > P (4) Q > P > R

Ans. **(1)**

Sol. P : $[FeF_6]^{-3} \Rightarrow 3d^5$ (WFL) $\Rightarrow n = 5$; $\mu = \sqrt{35}$

Q : $[V(H_2O)_6]^{+2} \Rightarrow 3d^3 \Rightarrow n = 3$; $\mu = \sqrt{15}$

R : $[Fe(H_2O)_6]^{+2} \Rightarrow 3d^6$ (WFL) $\Rightarrow n = 4$; $\mu = \sqrt{24}$

10. Electronic configuration of Nd(Z = 60) is :

Ans. $[Xe]\ 4f^4\ 6s^2$

11. **Statement-1:** $(NH_4)_2CO_3$ is basic.

Statement-2: Acidic nature of salt of WA & WB is dependent on K_a of WA & K_b of WB.

Ans. ($S_1 \rightarrow T$; $S_2 \rightarrow T$)

12. Number of electrons present in all the compound filled subshell having n = 4 and s = +1/2.

Ans. **(16)**

13. Consider following data :

$2HI(g) \rightarrow H_2(g) + I_2(g)$

	Experiment-1	Experiment-2	Experiment-3
HI(mole/litre)	0.005	0.01	0.02
Rate (mol L^{-1} s^{-1})	7.5×10^{-4}	3×10^{-3}	1.2×10^{-2}

Find order of reaction.

Ans. **(2)**

Sol. Rate = $K[HI]^x$ x = order

$$\frac{(Rate)_2}{(Rate)_1} = \left(\frac{[HI]_1}{[HI]_2}\right)^x$$

$$\frac{3\times10^{-3}}{7.5\times10^{-4}} = \left(\frac{0.01}{0.005}\right)^x$$

$4 = 2^x$

$\therefore x = 2$

14. If 3 moles of an ideal gas at 300 K expands isothermally from 30 dm^3 to 45 dm^3 against constant pressure of 80 K pascal then the amount of heat transfer is ___ joule.

Ans. **(1200)**

Sol. Process $\Rightarrow$ Isothermal, irreversible $\Rightarrow \Delta E = 0$

P_{ext} = Constant = 80 kPa

Expansion $V_1 = 30\ dm^3$ $V_2 = 45\ dm^3$

$\Delta E = 0 = q + W$

$q = -W$

$q = -[-P(V_2 - V_1)]$

$q = 80\ kPa\ [45\ dm^3 - 30\ dm^3]$

$= 80 \times 10^3\ Pa \times 15 \times 10^{-3}\ m^3$

$= 1200\ J$

15. The mass of silver (Ag = 108 gm/mole) displaces by a quantity of electricity which displaces 5600 ml of O_2 at STP will be :

Ans. **(108)**

Sol. mole × valency factor = mole × valency factor

$$\frac{W}{108}\times 1=\frac{5600}{22400}\times 4$$

$W = 108$ g

16. Which of the following has +4 oxidation state ?

(1) $H_2S_2O_7$ (2) H_2SO_3

Ans. **(2)**

Sol. $H_2S_2O_3$

$+2 + x - 6 = 0$

$x = +4$

17. Which halogen does not shows variable oxidation state ?

(1) F_2 (2) Cl_2 (3) Br_2 (4) I_2

Ans. **(1)**

Sol. F : Only (–1) in compounds

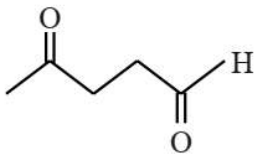
(∵ is not EN)

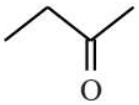
18. **Statement-1:** 4f & 5f series are written separately in periodic table in order to preserve principle of classification.

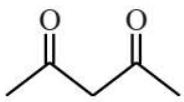
Statement-2: s-Block elements can be found on earth in pure form.

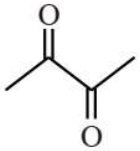
Ans. First statement is correct and second is not correct.

19. Which of the following compound is most acidic?

(1) 

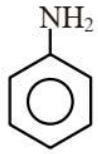
(2) 

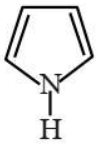
(3) 

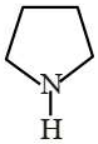
(4) 

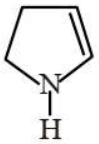
Ans. **(3)**

20. Which of the following is the strongest Bronsted base?

(1) 

(2) 

(3) 

(4) 

Ans. **(3)**

21. The correct statement regarding stereochemistry of S_N1 and S_N2 reaction is

(1) S_N1 – Racemisation
S_N2 – Retention

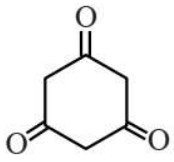
(2) S_N1 – Racemisation
S_N2 – Inversion

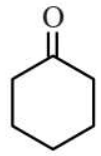
(3) S_N1 – Retention
S_N2 – Inversion

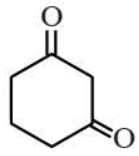
(4) S_N1 – Inversion
S_N2 – Retention

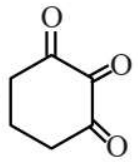
Ans. **(2)**

22. Which of the following has maximum enol content?

(1) 

(2) 

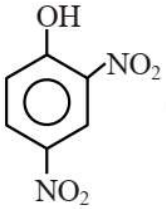
(3) 

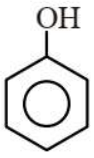
(4) 

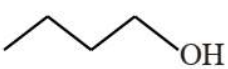
Ans. **(1)**

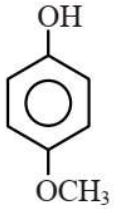
23. The correct order of acidic strength of the following compounds is

(I) 

(II) 

(III) 

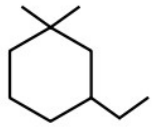
(IV) 

(V) 

(1) II > I > III > V > IV
(2) II > I > V > III > IV
(3) I > II > III > V > IV
(4) V > IV > III > I > II

Ans. (1)

24. The correct IUPAC name of following compound is



(1) 1,1-Dimethyl-3-ethyl cyclohexane

(2) 3-Ethyl-1,1-dimethyl cyclohexane

(3) 1-Ethyl-3,3-dimethyl cyclohexane

(4) 3,3-Dimethyl-1-ethyl cyclohexane

Ans. **(2)**

25. Cyclohexene is classified in

(1) Benzenoid aromatic (2) Alicyclic

(3) Benzenoid non aromatic (4) Acyclic

Ans. **(2)**

26. Which of the following is polar solvent

(1) CCl_4 (2) $CHCl_3$ (3) $CH_2=CH_2$ (4) CO_2

Ans. **(2)**

27. When nucleotide forms dimer the linkage present between is

(1) Disulphide linkage (2) Glycosidic linkage

(3) Phosphodiester linkage (4) Peptide linkage

Ans. **(3)**

MATHEMATICS

1. Find number of common terms in the two given series

4, 9, 14, 19……. up to 25 terms and

3, 9, 15, 21 …..up to 37 terms

(1) 4 (2) 7 (3) 5 (4) 3

Ans. **(1)**

Sol. 4, 9, 14, 19, ……. 124 $\rightarrow d_1 = 5$

3, 9, 15, 21 …….. 219 $\rightarrow d_2 = 6$

1st common term = 9 and common difference of common terms = 30

Common terms are 9, 39, 69, 99

4 common terms

2. Let $8 = 3 + \frac{3+p}{4} + \frac{3+2p}{4^2} + \ldots. \infty$, then p is

(1) 9 (2) $\frac{5}{4}$ (3) 3 (4) 1

Ans. **(1)**

Sol. $8 = 3 + \frac{3+p}{4} + \frac{3+2p}{4^2} + \ldots\ldots(i)$

multiply both sides by $\frac{1}{4}$, we get

$2 = \frac{3}{4} + \frac{3+p}{4^2} + \ldots.. (ii)$

Equation (i) – equation (ii)

$\Rightarrow 6 = 3 + \frac{p}{4} + \frac{p}{4^2} + \ldots\ldots$

$\Rightarrow 3 = \frac{p}{4\left(1-\frac{1}{4}\right)} \Rightarrow p = 9$

3. For $\frac{x^2}{25} + \frac{y^2}{16} = 1$, find the length of chord whose mid point is $P\left(1, \frac{2}{5}\right)$

(1) $\frac{\sqrt{1681}}{5}$ (2) $\frac{\sqrt{1481}}{5}$ (3) $\frac{\sqrt{1781}}{5}$ (4) $\frac{\sqrt{1691}}{5}$

Ans. **(4)**

Sol. By $T = S_1$

$$\Rightarrow \frac{x}{25}+\frac{y}{16}=\frac{1}{25}+\frac{4}{25}\cdot\frac{1}{16}$$

$$\Rightarrow \frac{x}{25}+\frac{y}{40}=\frac{4+1}{100}$$

$$\Rightarrow \frac{x}{25}+\frac{y}{40}=\frac{1}{20}$$

$$\Rightarrow 8x + 5y = 10$$

$$\Rightarrow \frac{x^2}{25}+\left(\frac{10-8x}{5}\right)^2\frac{1}{16}=1$$

$$\Rightarrow \frac{x^2}{25}+\frac{4}{25}\left(\frac{5-4x}{16}\right)^2=1$$

$$\Rightarrow x^2+\frac{(5-4x)^2}{4}=25$$

$$\Rightarrow 4x^2 + (5 - 4x)^2 = 100$$

$$\Rightarrow 20x^2 - 8x - 15 = 0$$

$$x_1 + x_2 = 2$$

$$x_1x_2 = \frac{-15}{4}$$

length of chord $= |x_1 - x_2|\sqrt{1+m^2}$

$$= \frac{\sqrt{1691}}{5}$$

4. If $f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3)$, then find $f'(10)$.

Ans. **(202)**

Sol. $f'(x) = 3x^2 + 2xf'(1) + f'(2)$

$f''(x) = 6x + 2f'(1)$

$f'''(3) = 6$

$f'(1) = -5$

$f''(2) = 2$

$\Rightarrow f'(10) = 300 + 20(-5) + 2$

$= 202$

5. Let $\int_0^1 \frac{dx}{\sqrt{x+3}+\sqrt{x+1}} = A + B\sqrt{2} + C\sqrt{3}$ then the value of $2A + 3B + C$ is

(1) 3 (2) 4 (3) 5 (4) 6

Ans. **(1)**

Sol. On rationalising

$$\int_0^1 \frac{(\sqrt{x+3}-\sqrt{x+1})}{2}dx$$

$$= \frac{2}{3.2}\left\{(x+3)^{3/2}-(x+1)^{3/2}\right\}_0^1$$

$$= \frac{1}{3}\{8-3\sqrt{3}-(2\sqrt{2}-1)\}$$

$$= \frac{1}{3}\{9-3\sqrt{3}-2\sqrt{2}\}$$

$$= \left(3-\sqrt{3}-\frac{2\sqrt{2}}{3}\right) : A=3, B=-\frac{2}{3}, C=-1$$

$\therefore$ $2A + 3B + C = 6 - 2 - 1 = 3$

6. If $|z-i| = |z-1| = |z+i|$, $z \in C$, then the numbers of z satisfying the equation are

(1) 0 (2) 1 (3) 2 (4) 4

Ans. **(2)**

Sol. z is equidistant from 1, i, & – i

only z = 0 is possible

$\therefore$ number of z equal to 1

7. If sum of coefficients in $(1-3x+10x^2)^n$ and $(1+x^2)^n$ is A and B respectively then

(1) $A^3 = B$ (2) $A = B^3$ (3) $A = 2B$ (4) $A = B$

Ans. **(2)**

Sol. $A = 8^n$ $B = 2^n$

(B) $\therefore$ $A = B^3$

8. Let $a_1, a_2, \ldots, a_{10}$ are 10 observations such that $\sum_{i=1}^{10} a_i = 50$ and $\sum_{i\neq j}^{10} a_i . a_j = 1100$, then their standard deviation will be

(1) $\sqrt{5}$ (2) $\sqrt{30}$ (3) $\sqrt{15}$ (4) $\sqrt{10}$

Ans. **(1)**

Sol. $(a_1 + a_2 + \ldots. + a_{10})^2 = 50^2$

$$\Rightarrow \sum a_1^2 + 2\sum_{i\neq j} a_i a_j = 2500$$

$$\Rightarrow \sum a_1^2 = 300$$

$$\sigma^2 = \frac{\sum a_i^2}{10} - \left(\frac{\sum a_i}{10}\right)^2$$

$\Rightarrow \sigma^2 = 5 \Rightarrow S.D = \sqrt{5}$

9. If $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ then

Statement-1 : $f(-x)$ is inverse of $f(x)$

Statement-2 : $f(x+y)=f(x)f(y)$

(1) Both are true (2) Both are false

(3) Only statement 1 is true (4) Only statement 2 is true

Ans. **(1)**

Sol. $f(x)f(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

$=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x-y) & 0\\ 0 & 0 & 1\end{bmatrix}$

$=f(x+y)$

$\therefore\ f(x)\,f(-x)=f(0)$

$=I$

10. If $a=\lim\limits_{x\to0}\dfrac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$ and $b=\lim\limits_{x\to0}\dfrac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ find $a\cdot b^3$

(1) 16 (2) 32 (3) –16 (4) 48

Ans. **(2)**

Sol. $a=\lim\limits_{x\to0}\dfrac{\sqrt{1+x^4}-1}{x^4\left[\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}\right]}$

$=\lim\limits_{x\to0}\dfrac{x^4}{x^4\left[\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}\right]\left[\sqrt{1+x^4}+1\right]}$

$=\dfrac{1}{2\sqrt{2}\times2}=\dfrac{1}{4\sqrt{2}}$

$b=\lim\limits_{x\to0}\dfrac{\sin^2 x}{(1-\cos x)}\left(\sqrt{2}+\sqrt{1+\cos x}\right)$

$=2\times\left(\sqrt{2}+\sqrt{2}\right)=4\sqrt{2}$

$\therefore\ ab^3=\left(4\sqrt{2}\right)^2=32$

11. If the minimum distance of centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ from any point on the parabola $y^2 = 4x$ is d, find d^2

Ans. **(20)**

Sol. Normal to parabola is $y = mx - 2m - m^3$

centre $(2, 8) \rightarrow 8 = 2m - 2m - m^3$

$\Rightarrow m = -2$

$\therefore$ p is $(m^2, -2m) = (4, 4)$

$\Rightarrow d^2 = 4 + 16 = 20$

12. If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$, $\vec{a} \times \vec{c} = \vec{b}$ & $\vec{a} \cdot \vec{c} = 3$ find $\vec{a} \cdot (\vec{c} \times \vec{b} - \vec{b} - \vec{c})$

(1) 24 (2) –24 (3) 18 (4) 15

Ans. **(1)**

Sol. $[\vec{a}\ \vec{c}\ \vec{b}] = (\vec{a} \times \vec{c}) \cdot \vec{b} = |\vec{b}|^2 = 27$

$\therefore$ we need $= 27 - 0 - 3 = 24$

13. Consider the line $L : 4x + 5y = 20$. Let two other lines are L_1 and L_2 which trisect the line L and pass through origin, then tangent of angle between lines L_1 and L_2 is

(1) $\frac{20}{41}$ (2) $\frac{30}{41}$ (3) $\frac{40}{41}$ (4) $\frac{10}{41}$

Ans. **(2)**

Sol. Let line L intersect the lines L_1 and L_2 at P and Q

$P\left(\frac{10}{3}, \frac{4}{3}\right), Q\left(\frac{5}{3}, \frac{8}{3}\right)$

$\therefore m_{OA} = \frac{2}{5}$

$m_{OQ} = \frac{8}{5}$

$\tan\theta = \left|\frac{\frac{8}{5} - \frac{2}{5}}{1 + \frac{16}{25}}\right|$

$= \left(\frac{6}{5} \times \frac{25}{41}\right)$

$= \frac{30}{41}$

14. If ${}^{n-1}C_r = (k^2 - 8)\,{}^{n}C_{r+1}$, then the range of 'k' is

(1) $k \in \left(2\sqrt{2}, 3\right]$ (2) $k \in \left(2\sqrt{2}, 3\right)$ (3) $k \in [2, 3)$ (4) $k \in \left(2\sqrt{2}, 8\right)$

Ans. (1)

Sol. ${}^{n-1}C_r = (k^2 - 8)\,\frac{n}{r+1}\,.\,{}^{n-1}C_r$

$\Rightarrow k^2 - 8 = \frac{r+1}{n}$

here $r \in [0, n-1]$

$\Rightarrow r + 1 \in [1, n]$

$\Rightarrow k^2 - 8 \in \left[\frac{1}{n}, 1\right]$

$\Rightarrow k^2 \in \left[8 + \frac{1}{n}, 9\right]$

$\Rightarrow k \in \left(2\sqrt{2}, 3\right]$

15. If $\alpha x + \beta y + 9\ln|2x + 3y - 8\lambda| = x + C$ is the solution of $(2x + 3y - 2)dx + (4x + 6y - 7)dy = 0$ then $\alpha + \beta + \gamma =$

(1) 18 (2) 19 (3) 20 (4) 21

Ans. **(1)**

Sol. Let $2x + 3y = t$

$\Rightarrow 2 + 3\frac{dy}{dx} = \frac{dt}{dx}$

Now $(t-2) + (2t-7)\left(\frac{dt}{dx} - 2\right) \times \frac{1}{3} = 0$

$\Rightarrow -\frac{(3t-6)}{2t-7} = \frac{dt}{dx} - 2$

$\Rightarrow \frac{dt}{dx} = \frac{t-8}{2t-7}$

$\Rightarrow \int \frac{2t-7}{t-8} dt = \int dx$

$\Rightarrow \int 2 + \frac{9}{t-8} dt = \int dx$

$\Rightarrow 2t + |\,9\ln|t-8| = x + C$

$\Rightarrow 2(2x + 3y) + 9\ln|2x + 3y - 8| = x + C$

$\alpha = 4, \beta = 6, \gamma = 8$

16. $f : N - \{1\} \to N$ and f(n) = highest prime factor of 'n', then f is

(1) one-one, onto (2) many-one, onto

(3) many-one, into (4) one-one, into

Ans. **(3)**

Sol. '4' is not image of any element $\Rightarrow$ into

$f(10) = 5 = f(15) \Rightarrow$ many-one

17. If P(X) represent the probability of getting a '6' in the X^{th} roll of a die for the first time. Also

$a = P(X = 3)$

$b = P(X \geq 3)$

$c = P\left(\frac{X \geq 6}{x > 3}\right)$, then $\frac{b+c}{a} = ?$

Ans. **(12)**

Sol. $P(X = 3) = \left(\frac{5}{6}\right)^2 . \frac{1}{6} = a$

$P(X \geq 3) = \left(\frac{5}{6}\right)^2 = b$

$P\left(\frac{X \geq 6}{X > 3}\right) = \left(\frac{5}{6}\right)^2 = c$

$\therefore \frac{b+c}{a} = \frac{2\left(\frac{5}{6}\right)^2}{\left(\frac{5}{6}\right)^2 . \frac{1}{6}} = 12$

18. If the angle between two vectors $\vec{a} = \alpha\hat{i} - 4\hat{j} - \hat{k}$ and $\vec{b} = \alpha\hat{i} + \alpha\hat{j} + 4\hat{k}$ is acute then find least positive integral value of α.

(1) 4 (2) 5 (3) 6 (4) 7

Ans. **(2)**

Sol. $\vec{a} . \vec{b} > 0$

$\Rightarrow \alpha^2 - 4\alpha - 4 > 0$

$\alpha < (2 - 2\sqrt{2})$ or $\alpha > (2 + 2\sqrt{2})$

19. If $S = \{1, 2, \ldots\ldots\ldots\ldots 10\}$ and M = P(S),

If ARB such that $A \cap B \neq \phi$ where $A \in M$, $B \in M$

Then

(1) R is reflexive and symmetric (2) Only symmetric

(3) Only reflexive (4) Symmetric and transitive

Ans. **(2)**

Sol. $\phi \cap \phi = \phi \quad \Rightarrow (\phi, \phi) \notin R \quad \Rightarrow$ not reflexive.

If $A \cap B \neq \phi \quad \Rightarrow B \cap A \neq \phi \Rightarrow$ Symmetric

If $A \cap B \neq \phi$ and $B \cap C \neq \phi \Rightarrow A \cap C = \phi$

for example $A = \{1, 2\}$

$B = \{2, 3\}$

$C = \{3,4\}$

20. If four points (0, 0), (1, 0), (0, 1), (2k, 3k) are concyclic, then k is

(1) $\frac{4}{13}$ (2) $\frac{5}{13}$ (3) $\frac{7}{13}$ (4) $\frac{9}{13}$

Ans. **(2)**

Sol. Equation of circle **is**

$x(x-1) + y(y-1) = 0$

$x^2 + y^2 - x - y = 0$

B(2k, 3k)

$\Rightarrow 4k^2 + 9k^2 - 2k - 3k = 0$

$\Rightarrow 13k^2 = 5k$

$\Rightarrow k = 0, \frac{5}{13}$

$\therefore k = \frac{5}{13}$

21. If f(x) is differentiable function satisfying $f(x) - f(y) \geq \log \frac{x}{y} + x - y$, then find $\sum_{N=1}^{20} f'\left(\frac{1}{N^2}\right)$

Ans. **(2890)**

Sol. Let $x > y$ Let $x < y$

$\lim_{y \to x} \frac{f(x)-f(y)}{x-y} \geq \frac{\log x - \log y}{x-y} + 1 \qquad \frac{f(x)-f(y)}{x-y} \leq \frac{\log x - \log y}{x-y} + 1$

$f'(x^-) \geq \frac{1}{x} + 1 \qquad f'(x^+) \leq \frac{1}{x} + 1$

$\Rightarrow f'(x^-) = f'(x^+)$ as f(x) is differentiable function

$f'(x) = \frac{1}{x} + 1$

$f'\left(\frac{1}{N^2}\right) = N^2 + 1$

$\sum_{N=1}^{20} f'\left(\frac{1}{N^2}\right) = \sum (N^2 + 1) = \frac{20 \times 21 \times 41}{6} + 20 = 2890$

22. Let $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ where $y(0) = 1$, $x(0) = 2$, and $x(t) = y(t)$, then t is

(1) $\frac{\ln 3}{a-b}$ (2) $\frac{\ln 2}{b-a}$ (3) $\frac{\ln 2}{a-b}$ (4) $\frac{\ln 3}{b-a}$

Ans. **(3)**

Sol. $\frac{dx}{dt} + ax = 0$

$\Rightarrow \ln x = -at + c$

$x(0) = 2 \Rightarrow c = \ln 2$

$\therefore x = 2e^{-at}$

$\frac{dy}{dt} + by = 0 \Rightarrow y = e^{-bt}$

$x(t) = g(t)$

$2e^{-at} = e^{-bt}$

$\Rightarrow t = \frac{\ln 2}{a-b}$

23. If H(a, b) is the orthocentre of ΔABC where A(1, 2), B(2,3) & C(3, 1), then find $\frac{36I_1}{I_2}$ if

$I_1 = \int_a^b x \sin(4x - x^2)\,dx$ and $I_2 = \int_a^b \sin(4x - x^2)\,dx$

Ans. **(72)**

Sol. ΔABC is isosceles

$\Rightarrow$ H lies on angle bisector passing through (3, 1) which is $x + y = 4$

$\therefore a + b = 4$

Now apply $(a + b - x)$ in I_1

$2I_1 = \int_a^b 4\sin(4x - x^2)\,dx$

$\Rightarrow 2I_1 = 4I_2$

$\Rightarrow \frac{I_1}{I_2} = 2$

$\therefore \frac{36I_1}{I_2} = 72$

24. $f(x)=\begin{cases} 2^{\frac{\sin(x-3)}{x-[x]}} & , \quad x>3 \\ -\dfrac{a(x^2-7x+12)}{b\,|x^2-7x+12|} & , \quad x<3 \\ b & , \quad x=3 \end{cases}$. Find number of ordered pairs (a, b) so that f(x) is continuous at x = 3

Ans. **(1)**

Sol. LHL = RHL = f(3)

$-\frac{a}{b}=2^1=b$

$\Rightarrow b=2$ and $a=-4$

$\Rightarrow (a,b)=(-4,2)$

25. Let $A=\begin{bmatrix}2&0&1\\1&0&0\\3&2&0\end{bmatrix}$, $B=[B_1\ B_2\ B_3]$ where B_1, B_2, B_3 are column matrices such that

$AB_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$, $AB_2=\begin{bmatrix}2\\0\\1\end{bmatrix}$, $AB_3=\begin{bmatrix}3\\2\\1\end{bmatrix}$

α = sum of diagonal elements of B

$\beta = |B|$, then find $|\alpha^3+\beta^3|$

Ans. **(1.125)**

Sol. $A^{-1}=\begin{bmatrix}0&1&0\\0&-\frac{3}{2}&\frac{1}{2}\\1&-2&0\end{bmatrix}$

$B_1=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $B_2=\begin{bmatrix}0\\\frac{1}{2}\\2\end{bmatrix}$, $B_3=\begin{bmatrix}2\\-\frac{5}{2}\\-1\end{bmatrix}$

$Tr(B)=-\frac{1}{2}$

$|B|=-1$

$\therefore a=-\frac{1}{2}, b=-1$

$|\alpha^3+\beta^3|=\frac{9}{8}=1.125$

26. If $\cos(2x) - a\sin x = 2a - 7$ has a solution for $a \in [p, q]$ and $r = \tan9° + \tan63° + \tan81° + \tan27°$, then p.q. r = ?

(1) $40\sqrt{5}$ (2) $32\sqrt{5}$ (3) $30\sqrt{5}$ (4) $48\sqrt{5}$

Ans. **(4)**

Sol. $2(\sin^2x - 4) + a(\sin x + 2) = 0$

$2(\sin x - 2) + a = 0$

$\Rightarrow a = 4 - 2\sin x$

$a \in [2, 6]$

Also, $r = \left(\tan 9° + \frac{1}{\tan 9°}\right) + \left(\tan 27° + 1\frac{1}{\tan 27°}\right)$

$= \frac{2}{\sin 18°} + \frac{2}{\sin 54°}$

$= \frac{2\times 4}{\sqrt{5}-1} + \frac{2\times 4}{\sqrt{5}+1}$

$= \frac{8\times 2\sqrt{5}}{4} = 4\sqrt{5}$

$\therefore pqr = 48\sqrt{5}$