JEE (Main)-2024 : Phase-1 (27-01-2024)-

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If the work function of a metal is 6.63 eV, then find its threshold frequency for photoelectric effect.

(1) 1.9×10^{15} Hz (2) 1.6×10^{15} Hz

(3) 2×10^{15} Hz (4) 1.2×10^{15} Hz

Answer (2)

Sol. $f_0 = \frac{\phi_0}{h} = \frac{6.63 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$

$= 1.6 \times 10^{15}$ Hz

2. If $\left(P - \frac{a}{V^2}\right)(V - b) = nRT$, where P, V, R and T are pressure, volume, universal gas constant and temperature, then $\frac{a}{b^2}$ has same dimensional formula as that of

(1) R (2) PV

(3) RT (4) P

Answer (4)

Sol. $\frac{a}{V^2} = P$

$a = PV^2$

$b = V$

$\left[\frac{a}{b^2}\right] = [P]$

3. **Statement-I** : Positive zero error is added in measurement.

Statement-II : Defect may occurs during manufacturing of measuring instruments.

(1) Statement-I is correct while statement-II is wrong

(2) Statement-I is wrong while statement-II is correct

(3) Both statements are wrong

(4) Both statements are correct

Answer (2)

Sol.
- Positive error always subtracted during measurement.
- Defect may occur during manufacturing of measuring device.

4. Find total kinetic energy of 1 mole of oxygen gas at 27°C. $\left[\text{Take } R = \frac{25}{3} \text{ J/mol-K}\right]$

(1) 6250 J (2) 3125 J

(3) 12500 J (4) 625 J

Answer (1)

Sol. ($f = 5$ for diatomic gas)

$K = \frac{5}{2}\mu RT = \frac{5}{2} \times 1 \times \frac{25}{3} \times 300$

$= 250 \times 25$

$= 6250$ J

5. If a current of 200 μA deflects the coil of moving coil galvanometer through 60°, then the current required to cause deflection through $\frac{\pi}{10}$ radians is

(1) 60 μA (2) 50 μA

(3) 20 μA (4) 150 μA

Answer (1)

Sol. $i \propto \theta$

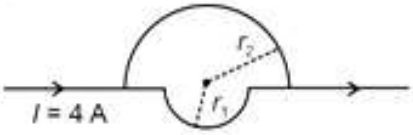
$\frac{i_1}{i_2} = \frac{\theta_1}{\theta_2}$

$\frac{200\ \mu A}{i} = \frac{\pi/3}{\pi/10}$

$i = \frac{200 \times 3}{10}\ \mu A$

$= 60\ \mu A$

6. Consider the following current carrying structure. Find the magnetic field at the centre. Given that $r_1 = 2\pi$ units and $r_2 = 4\pi$ units.



Assume current divides equally.

(1) 10^{-8} T (2) 5×10^{-8} T

(3) 10^{-7} T (4) 4×10^{-7} T

Answer (2)

Sol. $B = |B_1 - B_2|$

$$= \left| \frac{\mu_0\left(\frac{I}{2}\right)}{4r_1} - \frac{\mu_0\left(\frac{I}{2}\right)}{4r_2} \right|$$

$$= \frac{\mu_0 I}{2\times 4}\left[\frac{1}{4\pi}\right]$$

$$= \frac{4}{8}\times 10^{-7}\ \text{T} = 5\times 10^{-8}\ \text{T}$$

7. There exists a uniform electric field of $20\hat{i}$ N/C. A dipole of dipole moment $|\vec{p}| = 15$ C-m is placed at angle 30° with electric field. Torque on dipole is

(1) 250 Nm (2) 150 Nm

(3) 200 Nm (4) 100 Nm

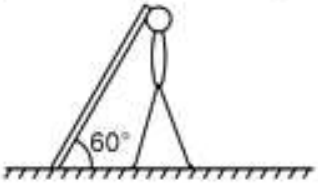
Answer (2)

Sol. $|\tau| = \text{P.E} \sin\theta$

$= 15 \times 20 \times \sin 30°$

$= 150$ Nm

8. A man holding a rod of mass m as shown in figure. Find weight of rod experienced by him.

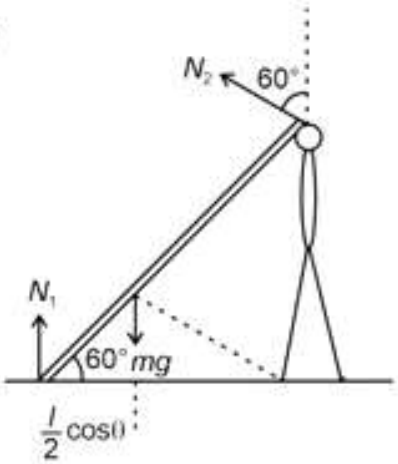


(1) $\frac{mg}{2}$ (2) $\frac{mg}{4}$

(3) $\frac{3mg}{2}$ (4) $\frac{mg}{3}$

Answer (2)

Sol.



$$mg\left(\frac{l}{2}\cos\theta\right) = N_2 \times l$$

$$mg \times \frac{1}{4} = N_2$$

$$N_2 = \frac{mg}{4}$$

9. If the primary side of a transformer is connected with 230 V, 50 Hz A.C supply and the ratio of number of turns of primary to the secondary winding is 10 : 1, load resistance at secondary coil is 46 Ω then power output of the secondary windings is

(1) 11.5 watt (2) 13 watt

(3) 16 watt (4) 15.6 watt

Answer (1)

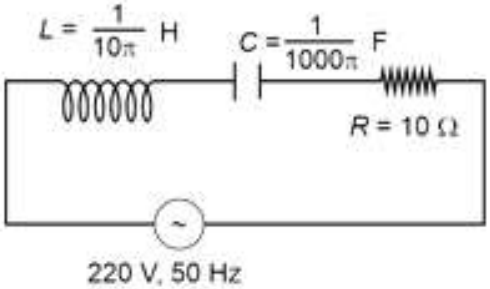
Sol. $\frac{V_i}{V_o} = \frac{N_i}{N_o} = 10$

$V_o = 23$ V

$$P_o = \frac{V_o^2}{R} = \frac{23\times 23}{46}$$

$P_o = 11.5$ W

10. Find the power factor of the given A.C circuit



(1) 0.75 (2) 0.5

(3) 1 (4) None of the above

Answer (3)

Sol. $X_L = \omega L = 100\pi \times \frac{1}{10\pi} = 10\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{100\pi \times \frac{1}{1000\pi}} = 10\ \Omega$$

$R = 10\ \Omega$

$$\cos\phi = \frac{R}{Z} = \frac{10}{\sqrt{(10-10)^2 + (10)^2}}$$

$= 1$

11. **Statement I :** Limiting friction depends on surface area.

Statement II : Kinetic friction depends on surface area.

(1) Statement I is true and statement II is false

(2) Statement II is true and statement I is false

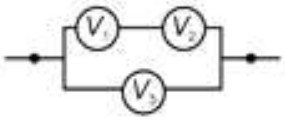
(3) Both statements are true

(4) Both statements are false

Answer (4)

Sol. The surface area does not affect frictional force.

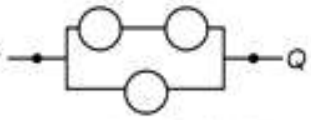
12. Three voltmeters are connected in a circuit as shown in diagram. Find correct relation among their readings (V_1, V_2 and V_3).



(1) $V_1 > V_2 = V_3$ (2) $V_1 + V_2 = V_3$

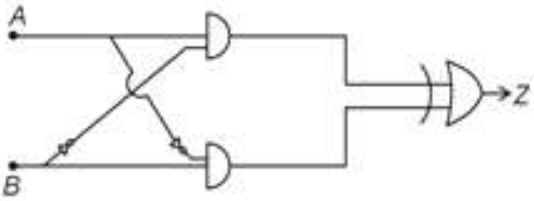
(3) $V_1 = V_2 = V_3$ (4) $V_1 + V_3 = V_2$

Answer (2)

Sol. 

Across points P & Q (V_1 & V_2) combined is in parallel to (V_3)

Therefore $V_{12} = V_3 \Rightarrow V_1 + V_2 = V_3$

13. 

Which is correct truth table for given circuit?

(1)

A	B	Z
0	0	1
0	1	1
1	0	1
1	1	0

(2)

A	B	Z
0	0	0
0	1	1
1	0	1
1	1	0

(3)

A	B	Z
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	Z
0	0	0
0	1	1
1	0	0
1	1	1

Answer (2)

Sol. $Z = AB' \oplus A'B$

$(AB'(A'B)' + (AB')' (A'B)$

$= AB'(A + B') + (A' + B)A'B$

$= AB' + AB' + A'B + A'B$

$= AB' + A'B$

$\Rightarrow$

A	B	Z
0	0	0
0	1	1
1	0	1
1	1	0

14. **Statement-1:** Work done by electrostatic force is zero if a charge is moving along an equipotential surface.

Statement-2: Equipotential surfaces are perpendicular to electric field.

(1) Statement-1 is true, statement-2 is false

(2) Statement-1 is false, statement-2 is true

(3) Both statements are false

(4) Both statements are true

Answer (4)

Sol. As electric field is perpendicular to the equipotential surface, there is no force along the equipotential surface.

15. In meter bridge, an unknown resistance X has specific resistance $S_1 = \frac{X\pi R^2}{l}$, where R is radius and l is length. If length and radius both are doubled, new specific resistance is :

(1) S_1

(2) $2S_1$

(3) $4S_1$

(4) $\frac{S_1}{4}$

Answer (1)

Sol. Specific resistance is specific to material.

$\Rightarrow$ Remains same.

16. In the given reaction, find value of Q value.

$${}_6C^{13} \longrightarrow {}_6C^{12} + {}_0n^1 + (Q - \text{value})$$

Given : mass of ${}_6C^{13} \Rightarrow x$

mass of ${}_6C^{12} \Rightarrow y$

mass of ${}_0n^1 \Rightarrow z$

(1) $(y + x - z)\,C^2$

(2) $(y + z - x)\,C^2$

(3) $(y + z + x)\,C^2$

(4) $(z + x - y)\,C^2$

Answer (2)

Sol. $\Rightarrow \quad \Delta m = (y + z - x)$

Q – value $= \Delta mC^2$

$= (y + z - x)\,C^2$

17. **Assertion** : Angular velocity of moon revolving about earth is more than angular velocity of earth revolving around sun.

Reason : Time taken by moon to revolve around earth is less than time taken by earth to revolve around sun.

(1) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(2) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)

(3) Assertion (A) is true and reason (R) is false

(4) Assertion (A) is false and reason (R) is true

Answer (1)

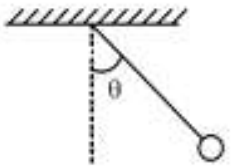
Sol. Time period of earth around sun is 365 days

then $\omega_1 = \dfrac{2\pi}{365 \text{ days}}$

Time period of moon around earth is 29 days

$\omega_2 = \dfrac{2\pi}{29 \text{ days}}$, so $\omega_2 > \omega_1$

18. A pendulum bob is released from angle θ with the vertical as shown in the figure. If it's acceleration at maximum amplitude is same as at mean position, find θ



(1) $\tan^{-1}\left(\sqrt{2}\right)$ (2) $2\tan^{-1}\left(\dfrac{1}{\sqrt{5}}\right)$

(3) $2\tan^{-1}\left(\dfrac{1}{2}\right)$ (4) $\tan^{-1}(2)$

Answer (3)

Sol. At max amplitude

$a_1 = g\sin\theta$

at mean position

$$a_2 = \frac{v^2}{l}$$

$\because \quad v^2 = 2gl(1 - \cos\theta)$

$\therefore \quad a_2 = 2g(1 - \cos\theta)$

$g\sin\theta = 2g\,(1 - \cos\theta)$

$$2g\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2g \times 2\sin^2\frac{\theta}{2}$$

$$\cos\frac{\theta}{2} = 2\sin\frac{\theta}{2}$$

$$\tan\frac{\theta}{2} = \frac{1}{2}$$

$$\theta = 2\tan^{-1}\left(\frac{1}{2}\right)$$

19. In a single slit diffraction pattern with slit width a and wavelength of light λ, find the angular position of first minima if screen distance is $D(D >> a)$

(1) $\dfrac{\lambda}{a}$ (2) $\dfrac{2\lambda}{a}$

(3) $\dfrac{3\lambda}{2a}$ (4) $\dfrac{3\lambda}{a}$

Answer (1)

Sol. For first minima

$$\sin\theta = \frac{\lambda}{a}, \quad \therefore \ \theta \approx \frac{\lambda}{a}$$

20. An atom of atomic number $Z = 50$ is having nuclear radius = 9×10^{-13} cm. Potential at the surface of the nucleus is

(1) 4×10^6 V

(2) 8×10^6 V

(3) 10^6 V

(4) 10^5 V

Answer (2)

Sol. $V = \frac{kQ}{r}$

$= \frac{9 \times 10^9 \times 50 \times 1.6 \times 10^{-19}}{9 \times 10^{-15}}$

$= 8 \times 10^6$ V

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A uniform ring and uniform solid sphere rolls down same inclined plane by same distance. If ratio of their translational kinetic energies is $\frac{7}{x}$ then x is

(Given mass and radius of ring and sphere are equal)

Answer (10.00)

Sol. $K = \frac{1}{2}mv^2 = \frac{1}{2}m(2as)$

$\Rightarrow a_r = \frac{g\sin\theta}{2}$

$\Rightarrow a_s = \frac{5}{7}g\sin\theta$

$\frac{K_r}{K_s} = \frac{\frac{1}{2}}{\frac{5}{7}} = \frac{7}{10}$

$\therefore \quad x = 10$

22. A bullet is fired into a fixed target. It loses $\frac{1}{3}$rd of its velocity after travelling 4 cm. It penetrates further $p \times 10^{-3}$ m before coming to rest. Find p.

Answer (32.00)

Sol. $v^2 - u^2 = 2as$

Let v_0 : initial

$\Rightarrow \left(\frac{2v_0}{3}\right)^2 - v_0^2 = 2(-a)\left(\frac{4}{100}\right)$

$\Rightarrow \frac{5}{9}v_0^2 = \frac{2a}{25}$ ...(i)

Also, $\frac{4v_0^2}{9} = 2 \times a \times \left(p \times 10^{-3}\right)$ ...(ii)

$\Rightarrow \frac{5}{4} = \frac{1000}{25 \times p}$

$\Rightarrow p = 32$

23. The expression for longest wavelength in Paschen series (for H atom) is $\frac{144R}{x}$. Find x. R is Rydberg's constant.

Answer (07.00)

Sol. Longest wavelength $\Rightarrow$ Minimum energy

$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right]$

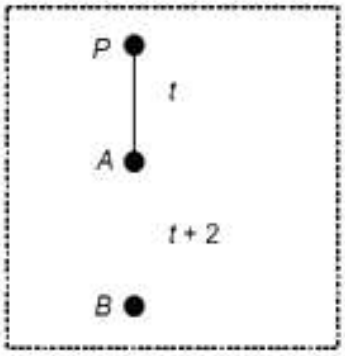
$= \frac{7R}{144} \Rightarrow \lambda = \frac{144R}{7}$

24. An object is dropped from certain height (from point P). It crosses 2 points A and B in interval of 2 seconds such that AB = 80 m. Find distance AP in meters.

(Take g = 10 m/s^2)

Answer (45)

Sol.



$$AB = \frac{1}{2}\varepsilon(t+2)^2 - \frac{1}{2}\varepsilon t^2$$

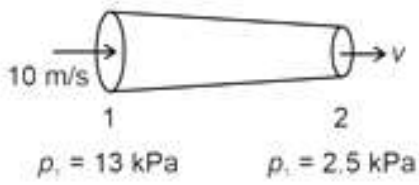
$$80 = 5(2t+2)(2)$$

$$4 = t + 1$$

$$t = 3$$

$$AP = \frac{1}{2}8t^2 = 5(3)^2 = 45\text{ m}$$

25. Pressures at ends of a horizontal pipe are given for water. Find speed v at end 2 if speed at end 1 is 10 m/s. (density of water = 1000 kg/m³). Find v (in m/s)



Answer (11.00)

Sol. $p_1 - p_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$

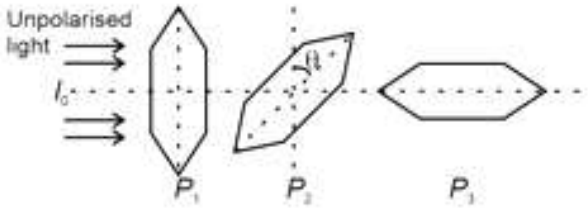
$$10.5\times10^3 = \frac{1}{2}\times10^3\left(v^2 - 10^2\right)$$

$$21 + 100 = v^2 \Rightarrow v = 11\text{ m/s}$$

$$p_1 - p_2 = 10.5\times10^3 = \frac{1}{2}\times10^3\left\{v_2^2 - v_1^2\right\}$$

$$1.21 = v_2$$

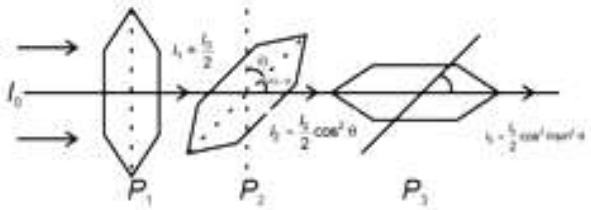
26. The diagram shows combination of polaroids.



Unpolarised light of intensity I_0 incident perpendicular to the axis of polaroid P_1, then angle θ for which maximum intensity passes through polaroid P_3.

Answer (45.00)

Sol.



$$I_3 = \frac{I_0}{2}\times\frac{4\sin^2\theta\cos^2\theta}{4}$$

$$= \frac{I_0}{8}(\sin2\theta)^2$$

For $I_3 \to$ max

$\sin2\theta = 1$

$2\theta = 90°$

$\theta = 45°$

27.

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The quantity which changes with temperature
   (1) Mole fraction
   (2) Mass percentage
   (3) Molarity
   (4) Molality

Answer (3)

Sol. Molarity of a solution depends upon temperature because volume of a solution is temperature dependent.

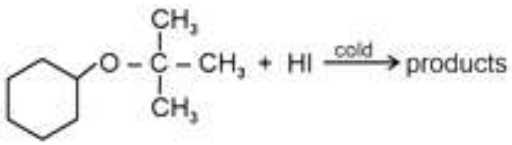
2. Which of the following cannot as an oxidising agent?
   (1) MnO_4^-
   (2) SO_4^{2-}
   (3) N^{3-}
   (4) BrO_3^-

Answer (3)

Sol. $N^{3-} \Rightarrow$ Because, it cannot further reduce itself to oxidise other.

$N^{3-} \Rightarrow -3 \Rightarrow$ lowest oxidation state

Others $\Rightarrow Mn^{+7}, S^{6+}, Br^{+5} \Rightarrow$ can be self reduced and oxidise others.

3. 

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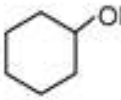
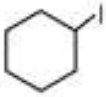
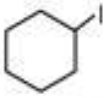
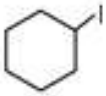
(1)  , $(CH_3)_3C-I$

(2)  , $(CH_3)_3C-I$

(3)  , $(CH_3)_3C-OH$

(4)  , $(CH_3)_3CH$

Answer (1)

Sol. During cleavage of ethers by cold HI, when one of the alkyl group is a tertiary group, the halide formed is a tertiary halide.

4. Identify the following species in which d^2sp^3 hybridisation is shown by central atom.
   (1) BrF_5
   (2) SF_6
   (3) $[Co(NH_3)_6]^{3+}$
   (4) $[PtCl_4]^{2-}$

Answer (3)

Sol. $Co^{3+} \Rightarrow 3d^6 \Rightarrow$ in presence of strong ligand NH_3

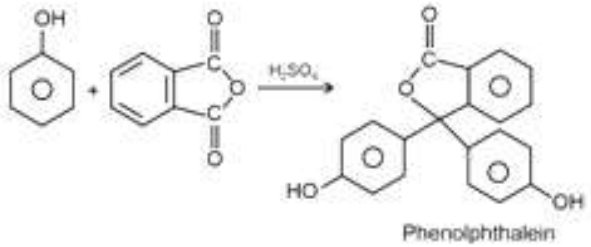
$3d^6 = t_{2g}^6 eg^0 \Rightarrow$ 2 inner d-orbitals are vacant forming d^2sp^3 hybridisation.

$BrF_5 \Rightarrow sp^3d^2$	$SF_6 \Rightarrow sp^3d^2$	$[PtCl_4]^{2-} \Rightarrow dsp^2$

5. Phenolic group can be identified by a positive
   (1) Lucas test
   (2) Carbylamine test
   (3) Phthalein test
   (4) Tollen's test

Answer (3)

Sol. Phenol on heating with phthalic anhydride in the presence of concentrated sulphuric acid forms a colourless condensation compound called phenolphthalein.



Phenolphthalein

6. Which structure of protein intact after coagulation of egg white on boiling

(1) Primary

(2) Secondary

(3) Tertiary

(4) Quaternary

Answer (1)

Sol. Primary structure of protein remain intact because it does not involve hydrogen bonding.

7. The molecular formula of second homologue in the homologous series of monocarboxylic acid is

(1) CH_3COOH

(2) CH_3CH_2COOH

(3) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-COOH$

(4) $CH_3CH_2CH_2-COOH$

Answer (1)

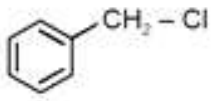
Sol. Homologous series of any functional groups differs in 1 CH_2 group, therefore;

1st Homologue $\Rightarrow$ HCOOH

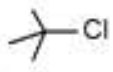
2nd Homologue $\Rightarrow$ CH_3COOH

8. Which of the following will not give S_N1.

(1) $CH_2=CH-CH_2-Cl$

(2) 

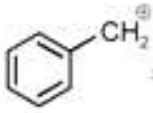
(3) $CH_3-CH=CH-Cl$

(4) 

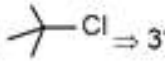
Answer (3)

Sol. For S_N1 reaction to occur, stable carbocation must from as intermediate.

(1) $CH_2=CH-\overset{\oplus}{C}H_2 \Rightarrow$ Allylic $\rightarrow$ Stable

(2)  $\Rightarrow$ Phenylic $\Rightarrow$ Stable

(3) $CH_3-CH=\overset{\oplus}{C}H \Rightarrow$ Unstable $\Rightarrow$ Vinylic $C^{\oplus}$

(4)  $\Rightarrow$ 3° Carbocation $\Rightarrow$ Stable

9. In which all have d^{10} configuration in their ground state?

(1) Cu, Zn, Cd, Ag

(2) Cd, Au, Hg, Ni

(3) Sc, Ti, Fe, Zn

(4) Fe, Cr, Co, Ni

Answer (1)

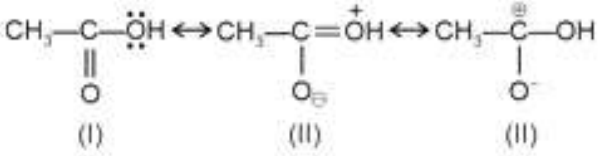
Sol. Cu : $3d^{10}4s^1$

Zn : $3d^{10}4s^2$

Cd : $4d^{10}5s^2$

Ag : $4d^{10}5s^1$

10. Compare the stability of resonating structures :


(I) (II) (II)

(1) I > II > III

(2) III > II > I

(3) I > III > II

(4) II > I > III

Answer (1)

Sol. Resonating structure which does not has any charge is most stable.

Now, if we compare II and III, in III octet of carbon is incomplete, therefore it is least stable.

11. Find out product (X)

$$Ph-CH=CH_2 \xrightarrow[(2)HBr\ (3)Mg,\ dry\ ether\ (4)HCHO]{(1)B_2H_6/THF/H_2O_2/OH^-} X(major)$$

(1) $Ph-\underset{\underset{OH}{|}}{CH}-CH_2-CH_3$

(2) $Ph-\underset{\underset{CH_2OH}{|}}{CH}-CH_3$

(3) $Ph-CH_2-\underset{\underset{CH_3}{|}}{CH}-OH$

(4) $Ph-CH_2-CH_2-CH_2-OH$

Answer (4)

Sol. $Ph-CH=CH_2 \xrightarrow{B_2H_6/THF/H_2O_2/OH^-} Ph-CH_2-CH_2-OH$

$\xrightarrow{HBr} Ph-CH_2-CH_2-Br$

$\xrightarrow[dry\ ether]{Mg} Ph-CH_2-CH_2-MgBr$

$\xrightarrow{HCHO} Ph-CH_2-CH_2-CH_2-OH$

12. The technique used for purification of steam volatile water immiscible substance is

(1) Fractional distillation

(2) Distillation under reduced pressure

(3) Steam distillation

(4) Simple distillation

Answer (3)

Sol. Substances which are steam volatile and are immiscible in water are separated by steam distillation.

13. If values of ΔH° & ΔS° for a process/reaction are 77.2 kJ & 48 J/k respectively. Then find value of $\log\frac{1}{k}$. Given : Temp is 300K

(1) 11

(2) 5

(3) 15

(4) 100

Answer (1)

Sol. $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$= 77.2 - 300 \times 48 \times 10^{-3}$

$= 77.2 - 14.4$

$= 62.8$ kJ

$\Delta G^\circ = -2.303\ RT \log k$

$$-\log k = \frac{62.8}{2.303 \times 8.314 \times 300 \times 10^{-3}} = 0.0109 \times 10^3$$

$= 10.9 \simeq 11$

$\log\frac{1}{k} = 11$ or option (1) is correct

14. Which of the following statement is correct?

(1) Ce^{4+} is oxidising agent

(2) Ce^{4+} is reducing agent

(3) Ce^{3+} has noble gas configuration

(4) Ce has stable configuration

Answer (1)

Sol. The E° value for Ce^{4+}/Ce^{3+} is +1.74 which explains that it is oxidising agent

15. **Statement I :** Oxygen is always present in –2 oxidation state.

Statement II : Stability of oxidation state of group16 for +4 and +6 decreases down the group.

(1) Both the statements are correct and Statement II is correct explanation of Statement I

(2) Both the statements are correct and Statement II does not support Statement I

(3) Statement I is correct and II is false

(4) Statement II is correct and Statement I is false

Answer (4)

Sol. Oxygen can show –2, –1, $-\frac{1}{2}$, +1, +2 and 0 oxidation state also.

Stability of +6 oxidation state decreases down the group due to inert pair effect.

16. Consider the following sequence of reactions

$$Cl-(CH_2)_4-Cl \xrightarrow{\text{Excess } NH_3} A \xrightarrow{NaOH} B + H_2O + NaCl$$

Find B

(1) $H_2N-(CH_2)_4-OH$

(2) $H_2N-(CH_2)_4-NH_2$

(3) $HO-(CH_2)_4-OH$

(4) $H_3\overset{+}{N}-(CH_2)_4-\overset{+}{N}H_3$

Answer (2)

Sol. $Cl-(CH_2)_4-Cl \xrightarrow{\text{Excess } NH_3} \underset{(A)}{ClH_3\overset{\oplus}{N}-(CH_2)_4-\overset{\oplus}{N}H_3Cl^{\ominus}}$

$\downarrow NaOH$

$\underset{(B)}{H_2N-(CH_2)_4-NH_2} + H_2O + NaCl$

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. For a first order reaction;

$t_{99.9} = x\ t_{50\%}$

Find out value of x

Answer (10)

Sol. $t_{99.9} = \dfrac{2.303}{k}\log\dfrac{100}{100-99.9}$

$t_{50} = \dfrac{2.303}{k}\log\dfrac{100}{100-50}$

$\dfrac{t_{99.9}}{t_{50}} = \dfrac{\log(1000)}{\log(2)} = \dfrac{3}{0.3} = 10$

22. If 84 g of NaOH (aq) is present in a 3 Molar Solution, find volume of solution (in mL)

Answer (700)

Sol. As we know

$$\text{Molarity} = \frac{\text{Moles of NaOH}}{\text{Volume of solution in L}}$$

$$3\ M = \frac{84}{\text{M.W. of NaOH} \times \text{Volume of Solution (L)}}$$

$$\text{Volume of Solution} = \frac{84}{40\times 3}\ L = 0.7\ L$$

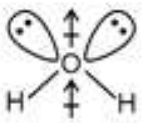
Volume of Solution in mL = 0.7 × 1000 = 700 mL

23. Number of non polar molecules given following.

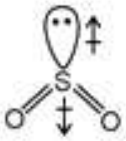
H_2O, SO_2, BF_3, H_2, $CHCl_3$

Answer (2)

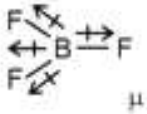
Sol. Molecules having zero dipole moment ($\mu = 0$) are non polar



$\mu \neq 0$



$\mu \neq 0$



$\mu = 0$

H – H

$\mu = 0$

$\mu \neq 0$

BF_3 and H_2 are non polar in nature

24. If longest wavelength for Paschen series in H-atom is $\dfrac{\alpha}{7R}$. Find out α

Answer (144)

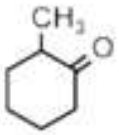
Sol. $\dfrac{1}{\lambda} = R(1)^2\left(\dfrac{1}{(3)^2} - \dfrac{1}{(4)^2}\right)$

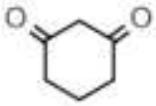
$\dfrac{1}{\lambda} = R\left(\dfrac{1}{9} - \dfrac{1}{16}\right)$

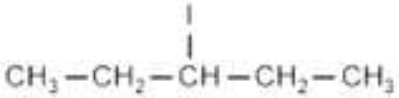
$\dfrac{1}{\lambda} = R\left(\dfrac{7}{9\times 16}\right)$

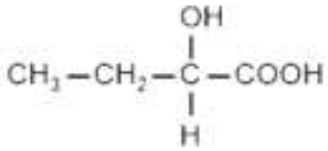
$\lambda = \dfrac{9\times 16}{7R} = \dfrac{144}{7R}$

25. How many compound(s) given below have chiral carbon?

(i) 

(ii) 

(iii) 

(iv) 

Answer (2)

Sol. Compound (i) and (iv) have chiral carbon.

26. If work function is 6.6 eV. The threshold frequency is $x \times 10^{14}$ Hz, Find x. ($h = 6.6 \times 10^{-34}$ J.S)

Answer (16)

Sol. $h\nu_0 = \phi$ (work function)

where ν_0 is threshold frequency

$h\nu_0 = 6.6 \times 1.6 \times 10^{-19}$ J

$$\nu_0 = \frac{6.6 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$\nu_0 = 1.6 \times 10^{15}$ Hz

$\nu_0 = 16 \times 10^{14}$

$\nu_0 = x \times 10^{14}$

x = 16

27. A hydrogen electrode is prepared by placing into a solution of pH = 3. The magnitude of electrode potential at 25°C is given by $t \times 10^{-2}$ V. Find out t (Nearest integer)

Answer (18)

Sol. $H_2 \rightarrow 2H^+ + 2e^-$

$$E_{cell} = 0 - \frac{0.0591}{2}\log(H^+)^2$$

$$= -\frac{0.0591}{2} \times 2 \times \log H^+$$

$= 0.0591 \times pH$

$= 0.1773$ V

$= 17.73 \times 10^{-2}$ V

t = 17.72

≈ 18

28. Magnetic moment of complex $[Pd(NH_3)_2Cl_2]$ will be

Answer (0)

Sol. In Pd^{2+}, NH_3 and Cl^- ligands behaves as SFL

$Pd^{+2} \Rightarrow 4d^8$

$[Pd(NH_3)_2Cl_2]$ is dsp^2 hybridised

Number of unpaired electron = 0

So Magnetic moment = 0

29. How many of the following given atomic number have noble gas configuration?

56, 57, 70, 80, 24

Answer (0)

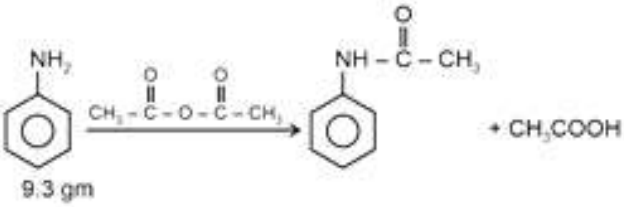
Sol. Atomic number of noble gases are

2, 10, 18, 36, 54, 86 and 118

None of the given atomic number belongs to noble gas

30. When 9.3 gm of aniline is reacted with acetic anhydride, mass of acetanilide obtained is m gm, find out value of 2m.

Answer (27)

Sol. 

9.3 gm

= 0.1 mole

0.1 mole

m = (0.1) × (135)

= 13.5 gm

2m = 27

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The integral $\int \frac{(x^8 - x^2)dx}{(x^{12} + 3x^6 + 1)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}$ is equal to

(1) $\frac{1}{3}\ln\left|\left(\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right)\right| + c$

(2) $\ln\left|\left(\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right)\right| + c$

(3) $\frac{1}{6}\ln\left|\left(\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right)\right| + c$

(4) $\frac{1}{9}\ln\left|\left(\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right)\right| + c$

Answer (1)

Sol. $\int \frac{x^2 - \frac{1}{x^4}dx}{\left(\left(x^3 + \frac{1}{x^3}\right)^2 + 1\right)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}$

Put $x^3 + \frac{1}{x^3} = t$

$\left(3x^2 - \frac{3}{x^4}\right)dx = dt$

$= \frac{1}{3}\int \frac{dt}{(t^2 + 1)\tan^{-1}t}$

Put $\tan^{-1}t = z$

$\frac{1}{1+t^2}dt = dz$

$= \frac{1}{3}\int \frac{dz}{z} = \frac{1}{3}\ln\left|\left(\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right)\right| + c$

2. If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions $\left[0, \frac{n\pi}{2}\right]$ for least value of $n \in N$, then $\sum_{K=1}^{n} \frac{K}{2^n}$ equal to

(1) $\frac{9}{2^9}$ (2) $\frac{91}{2^{13}}$

(3) $\frac{7}{2^7}$ (4) $\frac{11}{2^{12}}$

Answer (2)

Sol. $2\tan^2\theta - 5\sec\theta - 1 = 0$

$\Rightarrow 2(\sec^2\theta - 1) - 5\sec\theta - 1 = 0$

$\Rightarrow 2\sec^2\theta - 5\sec\theta - 3 = 0$

$\Rightarrow 2\sec^2\theta - 6\sec\theta + \sec\theta - 3 = 0$

$\Rightarrow (2\sec\theta + 1)(\sec\theta - 3) = 0$

$\sec\theta = 3$

$\Rightarrow \cos\theta = 1/3$

2 solutions in $[0, 2\pi]$

2 solutions in $[2\pi, 4\pi]$

2 solutions in $[4\pi, 6\pi]$

1 solution in $\left[6\pi, \frac{13\pi}{2}\right]$

$\Rightarrow n = 13$

$\sum_{K=1}^{13} \frac{K}{2^{13}}$

$\Rightarrow \frac{1}{2^{13}}(1 + 2 \ldots 13)$

$= \left(\frac{13 \cdot 14}{2}\right) \cdot \frac{1}{2^{13}}$

$= \frac{13 \cdot 7}{2^{13}} = \frac{91}{2^{13}}$

3. If $\frac{dy}{dx} = \frac{x + y - 2}{x - y}$ and $y(0) = 2$, find $y(2)$

(1) 0 (2) 2

(3) e (4) e^2

Answer (1)

Sol. Let $X = x - 1$ and $Y = y - 1$

$\Rightarrow \frac{dY}{dX} = \frac{X + Y}{X - Y}$

Now let $Y = vX$

$$\Rightarrow \frac{dY}{dX} = v + X\frac{dv}{dX}$$

$$v + X\frac{dv}{dX} = \frac{1+v}{1-v}$$

$$\Rightarrow \frac{(1-v)dv}{(1+v^2)} = \frac{dX}{X}$$

Integration

$$\tan^{-1}v - \ln\left(\sqrt{1+v^2}\right) = \ln|X| + c$$

Re-substituting

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \ln\sqrt{1+\left(\frac{y-1}{x-1}\right)^2} = \ln|(x-1)| + c$$

At $x = 0, y = 2$

$$\Rightarrow c = \frac{-\pi}{4} - \ln\sqrt{2}$$

at $x = 2$

$$\tan^{-1}(y-1) - \ln\sqrt{1+(y-1)^2} = \frac{-\pi}{4} - \ln\sqrt{2}$$

$$\Rightarrow y = 0, \text{ satisfy}$$

$$\Rightarrow y(2) = 0$$

4. $\int_0^{\pi} \frac{dx}{1-2a\cos x + a^2}$ is equal to

(1) $\frac{(1+a^2)\pi}{(1-a^2)^2}$ (2) $\frac{(1+a^2)\pi}{(1-a^2)}$

(3) $\frac{(1-a^2)\pi}{(1+a^2)}$ (4) $\frac{(1-a^2)\pi}{(1+a^2)^2}$

Answer (1)

Sol. $$I = \int_0^{\pi} \frac{1}{1-2(\cos x)a + a^2}dx$$

$$I = \int_0^{\pi} \frac{1}{1+2(\cos x)a + a^2}dx$$

$$2I = \int_0^{\pi}\left(\frac{1}{1-2a\cos x + a^2} + \frac{1}{1+2a\cos x + a^2}\right)dx$$

$$2I = \int_0^{\pi} \frac{2(1+a^2)}{(1+a^2)^2 - 4a^2\cos^2 x}dx$$

$$2I = 2\int_0^{\frac{\pi}{2}} \frac{2(1+a^2)}{(1+a^2)^2 - 4a^2\cos^2 x}dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{2(1+a^2)\sec^2 x}{(1+a^2)^2\sec^2 x - 4a^2}dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{2(1+a^2)\sec^2 x}{(1+a^2)^2(1+\tan^2 x) - 4a^2}dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{2(1+a^2)\sec^2 x}{(\tan^2 x)(1+a^2)^2 + (1-a^2)^2}dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{2}{1+a^2}\right)\sec^2 x}{\tan^2 x + \left(\frac{1-a^2}{1+a^2}\right)^2}dx$$

Now let $\tan x = t$

$\sec^2 x dx = dt$

$$I = \frac{2}{1+a^2}\int_0^{\infty} \frac{dt}{t^2 + \left(\frac{1-a^2}{1+a^2}\right)^2}$$

$$= \frac{2}{1+a^2} \times \frac{(1+a^2)^2}{(1-a^2)^2}\tan^{-1}\left[\frac{t(1+a^2)}{(1-a^2)}\right]_0^{\infty}$$

$$= \frac{2(1+a^2)}{(1-a^2)^2} \times \frac{\pi}{2}$$

$$= \frac{(1+a^2)\pi}{(1-a^2)^2}$$

5. The 20th term from the end of the progression

$20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots, -129\frac{1}{4}$ is

(1) –120 (2) –115

(3) –125 (4) –110

Answer (2)

Sol. $$\frac{-517}{4} = 20 + (n-1)\left(-\frac{3}{4}\right)$$

$\Rightarrow$ –517 = 80 + (–3n + 3)

$\Rightarrow$ –597 = –3n + 3

$\Rightarrow$ –600 = –3n

$\Rightarrow$ n = 200

n^{th} term from end is $(n - r + 1)^{th}$

$200 - 20 + 1 = 181^{th}$ term

$$a_{181} = 20 + (181 - 1)\left(-\frac{3}{4}\right)$$

$a_{181} = 20 - 135$

$a_{181} = -115$

6. An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability that the first draw gives all white balls and second draw gives all black balls is

(1) $\frac{2}{335}$ (2) $\frac{1}{495}$

(3) $\frac{5}{812}$ (4) $\frac{3}{715}$

Answer (4)

Sol. $P=\frac{{}^{6}C_{4}}{{}^{15}C_{4}}\cdot\frac{{}^{9}C_{4}}{{}^{11}C_{4}}$

$=\frac{15\times24}{15\cdot14\cdot13\cdot12}\times\frac{9\cdot8\cdot7\cdot6\times24}{24\cdot11\cdot10\cdot9\cdot8}$

$=\frac{1}{13\cdot7}\times\frac{7\cdot6}{10\cdot11}$

$=\frac{6}{13\cdot10\cdot11}$

$=\frac{3}{13\cdot5\cdot11}$

$=\frac{3}{715}$

7. Let $f:R-\left\{-\frac{1}{2}\right\}\to R$ and $g:R-\left\{-\frac{5}{2}\right\}\to R$ be defined as $f(x)=\frac{2x+3}{2x+1}$ and $g(x)=\frac{|x|+1}{2x+5}$ then the domain of the function $f(g(x))$ is

(1) R (2) $R-\left\{-\frac{5}{2}\right\}$

(3) $R-\left\{-\frac{1}{2},-\frac{5}{2}\right\}$ (4) $R-\left\{-\frac{1}{2}\right\}$

Answer (2)

Sol. $f(g(x))$

$\Rightarrow g(x)\neq-\frac{1}{2}$

$\frac{|x|+1}{2x+5}\neq\frac{-1}{2}$

(I) $x\geq0$

$\frac{x+1}{2x+5}=\frac{-1}{2}$

$2x+2=-2x-5$

$4x=-7$

$x=\frac{-7}{4}$ (Rejected)

(II) $x<0$

$\frac{-x+1}{2x+5}=\frac{-1}{2}$

$-2x+2=-2x-5$

$2=-5$ (not possible)

$\Rightarrow$ Domain of $f(g(x))$ = domain of $g(x)$

$D_{fog}=R-\left\{\frac{-5}{2}\right\}$

8. Considering the principal values of inverse trigonometric functions, the positive real values of 'x' satisfying $\tan^{-1}x+\tan^{-1}(2x)=\frac{\pi}{4}$ is

(1) $\frac{\sqrt{5}-1}{2}$ (2) $\frac{\sqrt{17}+3}{4}$

(3) $\frac{\sqrt{17}-3}{4}$ (4) $\frac{\sqrt{5}+1}{2}$

Answer (3)

Sol. $\tan^{-1}x+\tan^{-1}2x=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{3x}{1-2x^2}\right)=\frac{\pi}{4}$

$\Rightarrow \frac{3x}{1-2x^2}=1$

$\Rightarrow 2x^2+3x-1=0$

$\Rightarrow x_1,x_2=\frac{-3\pm\sqrt{9+8}}{4}$

$\Rightarrow x_1,x_2=\frac{-3\pm\sqrt{17}}{4}$

$\therefore\ x_1=\frac{\sqrt{17}-3}{4}>0$

9. Let R be the interior region between the lines $3x-y+1=0$ and $x+2y-5=0$ containing the origin. The set of all values of 'a' for which the points $(a^2, a+1)$ lies in R is

(1) $(-\infty,-1)\cup(3,\infty)$

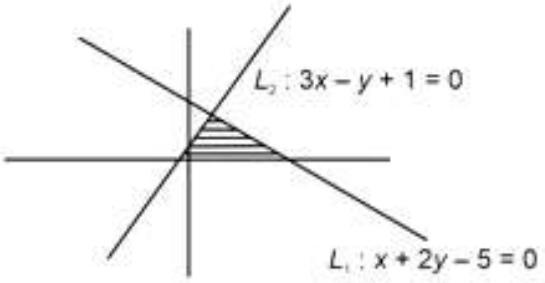
(2) $(-3,0)\cup\left(\frac{1}{3},1\right)$

(3) $(-\infty,-1)\cup\left(0,\frac{1}{3}\right)$

(4) $(-\infty,-2)\cup\left(0,\frac{1}{3}\right)$

Answer (2)

Sol.



R is the shaded region where $(a^2, a+1)$ should lie.

For line L_1,

$\therefore \quad a^2 + 2(a+1) - 5 < 0$

$a^2 + 2a - 3 < 0$

$(a+3)(a-1) < 0$

$\Rightarrow a \in (-3, 1) \qquad \ldots (1)$

Also, for line L_2

$3a^2 - a - 1 + 1 > 0$

$3a^2 - a > 0$

$a(3a-1) > 0$

$$a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right) \qquad \ldots (2)$$

$\therefore (1) \cap (2)$

$$a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$$

10. If $f(x) = 6x - x^2$, $x \in [0, 2]$ and

$$g(x) = \begin{cases} \min f(t),\ 0 \le t \le x,\ x \in [0, 1] \\ 3 + x,\ x \in [1, 2] \end{cases}$$

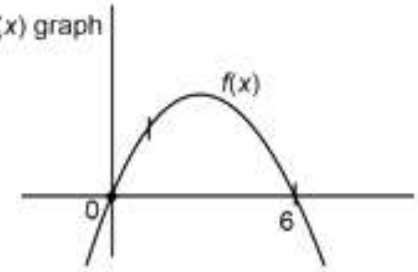
Then number of points where $g(x)$ is not differentiable is

(1) 1
(2) 0
(3) 2
(4) 3

Answer (1)

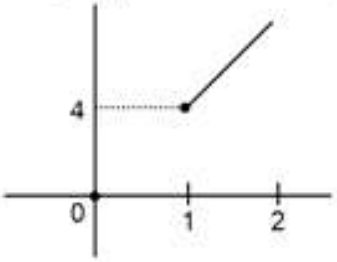
Sol. $$g(x) = \begin{cases} \min (f(t)),\ 0 \le t \le x,\ x \in [0, 1] \\ 3 + x,\ x \in [1, 2] \end{cases}$$

$f(x)$ graph



For $x \in [0, 1]$, $\min f(t) = 0$

For $x \in [1, 2]$, $f(t) = 3 + x$



Number of points of non-differentiability is 1

11. If $H : \frac{x^2}{16} - \frac{y^2}{9} = 1$ and $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$.

Ellipse passes through the foci of the hyperbola and $e_1 \cdot e_2 = 1$ (where e_1, e_2 are the eccentricities of hyperbola and ellipse, respectively). The length of the chord of ellipse passing through (0, 2) is equal to

(1) $\frac{5\sqrt{10}}{3}$ (2) $\frac{10\sqrt{5}}{3}$

(3) $2\sqrt{5}$ (4) $2\sqrt{10}$

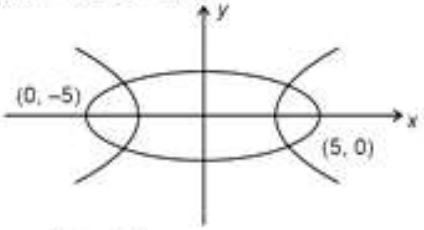
Answer (2)

Sol. $$e_1^2 = 1 + \frac{9}{16} \Rightarrow e_1 = \frac{5}{4}$$

$$\therefore \quad e_2 = \frac{4}{5}$$

Foci of hyperbola = $(\pm 5, 0)$

$2a = 10 \Rightarrow a = 5$



$$1 - \frac{b^2}{a^2} = \frac{16}{25}$$

$\Rightarrow \quad b = 3$

$$E : \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$\frac{x^2}{25} + \frac{4}{9} = 1 \ldots (y = 2)$$

$$\Rightarrow \quad \frac{x^2}{25} = \frac{5}{9}$$

$$x = \pm \frac{5\sqrt{5}}{3}$$

$$\therefore \quad x_1 = -\frac{5\sqrt{5}}{3}, \qquad x_2 = \frac{5\sqrt{5}}{3}$$

$$\therefore \quad l = \frac{10\sqrt{5}}{3}$$

Option (2) is correct.

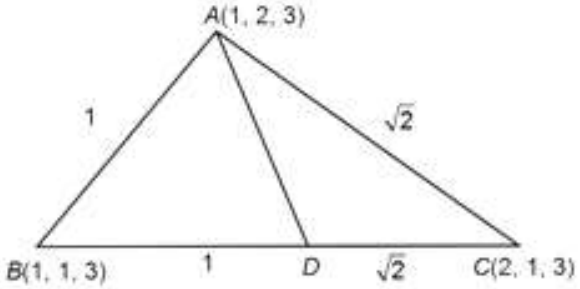
12. The position vector of vertices A, B, C of Δ are $\hat{i}+2\hat{j}+3\hat{k}$, $\hat{i}+\hat{j}+3\hat{k}$, $2\hat{i}+\hat{j}+3\hat{k}$ respectively. Let l is the length of angle bisector of $\angle BAC$, then the value of l^2 is

(1) $4+2\sqrt{2}$ (2) $4-2\sqrt{2}$

(3) $2+2\sqrt{2}$ (4) $2-2\sqrt{2}$

Answer (2)

Sol.



$$\frac{BD}{DC}=\frac{1}{\sqrt{2}}$$

$$D\left(\frac{2+\sqrt{2}}{\sqrt{2}+1},1,3\right)$$

Now $AD: l=\sqrt{\left(\frac{2+\sqrt{2}}{\sqrt{2}+1}\right)^2+1}$

$$l^2=\left(\frac{1}{\sqrt{2}+1}\right)^2+1$$

$$=\left(\sqrt{2}-1\right)^2+1$$

$$=4-2\sqrt{2}$$

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If for two sets A and B, $n(A)=m$ and $n(B)=n$ and (Number of subsets of A – Number of subsets of B) = 56, then find value of $(2m+n)$.

Answer (15)

Sol. $n(A)=m$, then number of subsets of $A=2^m$

$n(B)=n$, then number of subsets of $B=2^n$

given that

$2^m-2^n=56$

$2^m-2^n=2^6-2^3$

From here, $m=6$

$n=3$

$2m+n=2\times6+3=15$

22. If A is a 2×2 matrix and I is an identity matrix of order 2 & $|A-\lambda I|=0$ gives value of λ as –1 & 3 then trace of A^2 is equal to _____

Answer (10)

Sol. Given $|A-\lambda I|=0$

Value of 'λ' are –1 and 3

Now value of 'λ' of A^2 is equal to λ^2

$\therefore$ Let $\lambda_1=(-1)^2=1$

$\lambda_2=(3)^2=9$

Trace of $A^2=1+9$

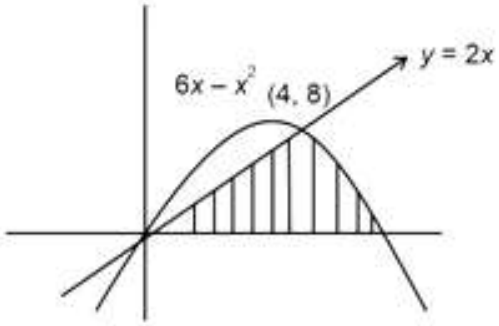
$=10$

$\therefore$ Answer is 10

23. The area bounded by $0\le y\le\min\{2x,6x-x^2\}$ and x-axis is A. Then $12A$ is

Answer (304)

Sol. $\min\{6x-x^2, y=2x\}$



$$\text{Area}=\frac{1}{2}\times4\times8+\int_4^6(6x-x^2)dx$$

$$=16+\left(3x^2-\frac{x^3}{3}\right)_4^6$$

$$A = 16 + \frac{28}{3}$$

$12A = (12 \times 16 + 28 \times 4)$

= 304 square unit

24. If the line $x + y = 0$ is tangent to the circle $(x - \lambda)^2 + (y - \beta)^2 = 50$, then $(\lambda + \beta)^2 =$

Answer (100.00)

Sol. Perpendicular distance from centre = radius

$$\Rightarrow \left|\frac{\lambda + \beta}{\sqrt{2}}\right| = \sqrt{50}$$

$$|\lambda + \beta| = \sqrt{100}$$

$(\lambda + \beta)^2 = 100$

25. If $f(x) = \int_0^x g(t) \ln\left(\frac{1-t}{1+t}\right) dt$ and g is odd continuous function and $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(f(x) + \frac{x^2 \cos x}{(1+e^x)}\right) dx = \frac{\pi^2}{\alpha^2} - \alpha$ then α is

Answer (2)

Sol. $\therefore \quad f(x) = \int_0^x g(t) \ln\left(\frac{1-t}{1+t}\right) dt$

$$\therefore \quad f'(x) = g(x). \ln\left(\frac{1-x}{1+x}\right)$$

Here $f'(x)$ is even since $g(x)$ and $\ln\left(\frac{1-x}{1+x}\right)$ both are odd

Hence $f(x)$ is odd function $\Rightarrow f(x) + f(-x) = 0$

Now $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(f(x) + \frac{x^2 \cos x}{(1+e^x)}\right) dx$

$$= \int_0^{\frac{\pi}{2}} \left\{f(x) + f(-x) + x^2 \cos x \left(\frac{1}{1+e^x} + \frac{1}{1+e^{-x}}\right)\right\} dx$$

$$= \int_0^{\frac{\pi}{2}} x^2 \cos x \, dx$$

$$= \left[x^2 \sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2x \sin x \, dx$$

$$= \frac{\pi}{4} - 2\left\{\left[-x \cos x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -\cos x \, dx\right\}$$

$$= \frac{\pi^2}{4} - 2$$

Given that $\frac{\pi^2}{\alpha^2} - \alpha = \frac{\pi^2}{4} - 2$

$\therefore \quad \alpha = 2$

26. If α and β are the roots of the equation $x^2 - x - 1 = 0$ and $S_n = 2024\,\alpha^n + 2024\,\beta^n$ then S_3 is equal to

Answer (8096)

Sol. $x^2 - x - 1 = 0 \nearrow \alpha \searrow \beta \Rightarrow \begin{matrix} \alpha + \beta = 1 \\ \alpha\beta = -1 \end{matrix}$

$\Rightarrow \alpha^2 = \alpha + 1$

$\beta^2 = \beta + 1$

$S_3 = 2024\alpha^3 + 2024\beta^3$

$= 2024\alpha\,[\alpha + 1] + 2024\beta[\beta + 1]$

$= 2024\alpha + 2024\beta + 2024\alpha^2 + 2024\beta^2$

$= 2024\alpha + 2024\beta + 2024\,(\alpha + 1) + 2024\,(\beta + 1)$

$= 4048\,(a + b) + 4048$

$= 4048 + 4048 = 8096$

27. If the mean of 15 observations is 12 and standard deviation is 3. If 12 is replaced by 10 (in data) then the new mean is μ and variance is σ^2 then what is the value of 15 $(\mu + \mu^2 + \sigma^2)$

Answer (2429)

Sol. Given mean is 12 and $n = 15$

So take data as

$x_1, x_2, \ldots, x_{14}, 12$

so $\frac{x_1 + x_2 + \ldots + x_{14} + 12}{15} = 12$

$x_1 + x_2 + \ldots + x_{14} = 168$

Now in place of 12, we need to write 10,

$$\frac{168 + 10}{15} = \frac{178}{15} = \bar{x}_{new}$$

And also,

$$9 = \frac{\sum x_i^2}{15} - (144)$$

$$\Rightarrow \quad \sum x_i^2 = 153 \times 15$$

$$\Rightarrow \quad \sum_{i=1}^{n} x_i^2 - 144 + 100 = 2251$$

$$\text{Variance} = \frac{2251}{15} - \left(\frac{178}{15}\right)^2 = \sigma^2$$

$$15(\mu + \mu^2 + \sigma^2) = 15\left(\frac{2251}{15} + \frac{178}{15}\right)$$

$$= 2429$$

28. If $\lim\limits_{x \to 0} \dfrac{3 + \alpha \sin x + \beta \cos x + \ln(1 - x)}{3 \tan^2 x} = \dfrac{1}{3}$.

Then $2\alpha - \beta$ is equal to

Answer (5)

Sol. Using expansions,

$$\lim_{x \to 0} \left(\frac{3 + \alpha\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\right) + \beta\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\right) + \left(-x - \frac{x^2}{2} - \frac{x^3}{3}\right)}{3\left(x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots\right)^2} \right)$$

$$\lim_{x \to 0} \frac{(3 + \beta)x^0 + x^1(\alpha - 1) + x^2\left[\frac{-\beta}{2} - \frac{1}{2}\right]}{3x^2\left(1 + \frac{x^2}{3} + \frac{2x^4}{15} + \ldots\right)^2} = \frac{1}{3}$$

$\beta = -3$

$\alpha = 1$

$$\frac{-(\beta + 1)}{2 \times 3} = \frac{1}{3} \Rightarrow (\beta + 1) = -2$$

$\Rightarrow \quad \beta = -3$

$\Rightarrow \quad 2\alpha - \beta = 2 - (-3) = 5$

29.

30.