# JEE (Main)-2024 : Phase-1 (01-02-2024)-Morning

## PHYSICS

### SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer:**

1. The dimensions of angular impulse is equal to

(1) $[M^1L^2T^{-1}]$ (2) $[M^1L^2T^1]$

(3) $[M^1L^2T^2]$ (4) $[M^1L^1T^{-1}]$

**Answer (1)**

**Sol.** Angular impulse = Change in angular momentum

$[J] = [mvr]$

$[J] = [M^1L^2T^{-1}]$

2. A vernier caliper has 10 main scale divisions coinciding with 11 vernier scale divisions. 1 main scale division equals 5 mm. The least count of the device is

(1) $\frac{1}{2}$ mm (2) $\frac{5}{12}$ mm

(3) $\frac{5}{11}$ mm (4) 0.3 mm

**Answer (3)**

**Sol.** 10 M = 11 V

$\Rightarrow 1\,V = \frac{10}{11} \times 5$ mm

$\Rightarrow LC = |M - V|$

$= \frac{5}{11}$ mm

3. On increasing temperature, the elasticity of a material

(1) Increases

(2) Decreases

(3) Remains constant

(4) May increase or decrease

**Answer (2)**

**Sol.** $E = \frac{\text{Stress}}{\text{Strain}}$

As temperature increases, strain increases

$\therefore$ Elasticity decreases

4. Determine the lowest energy of photon emitted in Balmer series of hydrogen atom.

(1) 10.02 eV

(2) 1.88 eV

(3) 1.65 eV

(4) 2.02 eV

**Answer (2)**

**Sol.** For $3 \rightarrow 2$ transitions

$\Delta E = 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$

$= 13.6 \times \frac{5}{36}$

= 1.88 eV

5. de Broglie wavelength of proton = $\lambda$ and that of an $\alpha$ particle is $2\lambda$. The ratio of velocity of proton to that of $\alpha$ particle is :

(1) 8 (2) $\frac{1}{8}$

(3) 4 (4) $\frac{1}{4}$

**Answer (1)**

**Sol.** $\lambda = \frac{h}{p}$

$\Rightarrow \lambda = \frac{h}{mv_p}$

and $2\lambda = \frac{h}{4mv_\alpha}$

$\Rightarrow \frac{1}{2} = \frac{4v_\alpha}{v_p}$

$\Rightarrow \frac{v_p}{v_\alpha} = 8$

6. 2 moles of monoatomic gas and 6 moles of diatomic gas are mixed. Molar specific heat, for constant volume, of mixture shall be ($R$ is universal gas constant)

(1) 1.75$R$ (2) 2.25$R$

(3) 2.75$R$ (4) 2.50$R$

**Answer (2)**

**Sol.** $(C_V)_{mix} = \left(\frac{2\times\frac{3}{2}+6\times\frac{5}{2}}{2+6}\right)R$

$= \frac{(3+15)R}{8} = \frac{9}{4}R$

7. A gas undergoes a thermodynamic process from state ($P_1$ $V_1$ $T_1$) to state ($P_2$, $V_2$, $T_2$). For the given process if $PV^{\frac{3}{2}}$ = constant, find the work done by the gas.

(1) $\frac{(P_2V_2 - P_1V_1)}{2}$ (2) $\frac{(P_1V_1 - P_2V_2)}{2}$

(3) $\frac{3}{2}(P_1V_1 - P_2V_2)$ (4) $2(P_1V_1 - P_2V_2)$

**Answer (4)**

**Sol.** $W = \frac{P_1V_1 - P_2V_2}{\alpha - 1}$

$= \frac{P_1V_1 - P_2V_2}{\left(\frac{3}{2}-1\right)}$

$= 2(P_1V_1 - P_2V_2)$

8. For measuring resistivity, the relation $R = \rho\frac{l}{A} = \frac{\rho l}{\pi r^2}$ is used. Percentage error in resistance ($R$), in length ($l$) and in radius ($r$) are given $x$, $y$ and $z$ respectively. Find percentage error in resistivity $\rho$.

(1) $x + y + 2z$ (2) $x + 2y + z$

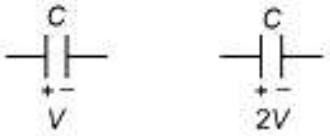
(3) $\frac{x}{2} + y + z$ (4) $x + 2z - y$

**Answer (1)**

**Sol.** $\frac{\Delta\rho}{\rho} = \frac{\Delta R}{R} + \frac{2\Delta r}{r} + \frac{\Delta l}{l}$

$= x + 2z + y.$

9. Two capacitors are charged as shown. When both the positive terminals and negative terminals of capacitors are connected the energy loss will be




(1) $\frac{1}{2}CV^2$ (2) $\frac{3}{4}CV^2$

(3) $\frac{1}{4}CV^2$ (4) $2CV^2$

**Answer (3)**

**Sol.** $V_C = \frac{CV + 2CV}{2C} = \frac{3V}{2}$

$\therefore$ Energy loss = $\frac{1}{2}CV^2 + \frac{1}{2}C(2V)^2 - \frac{1}{2}2C\left(\frac{3V}{2}\right)^2$

$= \frac{1}{4}CV^2$

10. A moving coil galvanometer has resistance 50 Ω and full deflection current is 5 mA. The resistance needed to convert this galvanometer into voltmeter of range 100 volt is

(1) 19550 Ω (2) 18500 Ω

(3) 19850 Ω (4) 18760 Ω

**Answer (1)**

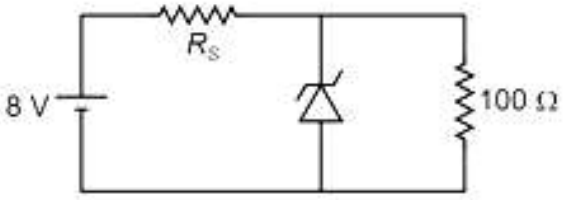
**Sol.** $I_g(G + R) = 100$ V

~~5~~ × $10^{-3}(50 + R) = ~~100~~^{20}$

$50 + R = 20000$

$R = 19550$ Ω

11. In the voltage regulator circuit shown below, the reverse breakdown voltage of zener diode is 5 V and power dissipated across it is 100 mW. Find $R_S$




(1) 120 Ω (2) 250 Ω

(3) 1000 Ω (4) 1500 Ω

**Answer (1)**

**Sol.** $i_{1000\,\Omega} = 5$ mA

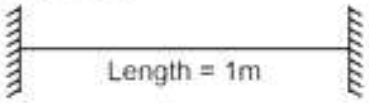
$$i_Z = \frac{P}{V_Z} = 20 \text{ mA}$$

$$\therefore \quad i_R = 25 \text{ mA}$$

$V_R = 3$ V

$$\therefore \quad R = \frac{3}{25} \times 10^3 = 120\ \Omega$$

12. Two strings are identical and fixed at both ends with tension 6 N each. If the tension in one string fixed at both end is changed from 6 N to 52 N, then find beats frequency.




Linear mass density = 1 kg/m

(1) 2.38 Hz (2) 3.25 Hz

(3) 2.75 Hz (4) 5.25 Hz

**Answer (1)**

**Sol.** $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$$f_1 = \frac{1}{2L}\sqrt{\frac{T_1}{\mu}}$$

$$f_2 = \frac{1}{2L}\sqrt{\frac{T_2}{\mu}}$$

$$\text{Beats frequency} = \Delta f = f_2 - f_1 = \frac{1}{2L}\left(\sqrt{\frac{52}{\mu}} - \sqrt{\frac{6}{\mu}}\right)$$

$$= \frac{1}{2}\left(\sqrt{52} - \sqrt{6}\right)$$

$$= \frac{1}{2}(7.21 - 2.45)$$

= 2.38 Hz

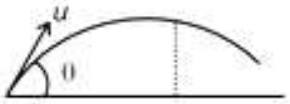
13. A particle is moving in a circle of radius $R$ in time period of $T$. This moving particle is projected at angle $\theta$ with horizontal & attains a maximum height of $4R$. Angle $\theta$ can be given as ($g$ is acceleration due to gravity)

(1) $\sin^{-1}\left(\frac{T}{2\pi}\sqrt{\frac{2g}{R}}\right)$ (2) $\sin^{-1}\left(\frac{T}{\pi}\sqrt{\frac{g}{R}}\right)$

(3) $\sin^{-1}\left(\frac{T}{\pi}\sqrt{\frac{2g}{R}}\right)$ (4) $\sin^{-1}\left(T\sqrt{\frac{2g}{R}}\right)$

**Answer (3)**

**Sol.** $\frac{2\pi R}{T} = u$

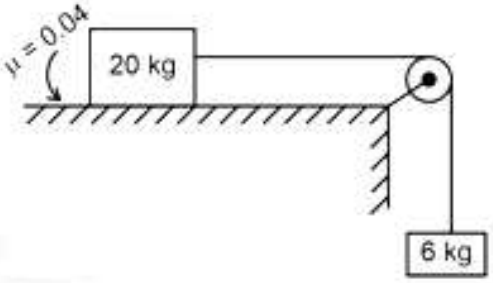



$$\frac{u^2 \sin^2\theta}{2g} = 4R$$

$$\frac{4\pi^2 R^2}{T^2\, 2g}\sin^2\theta = 4R$$

$$\sin^2\theta = \frac{2gT^2}{\pi^2 R} = \left(\frac{T}{\pi}\sqrt{\frac{2g}{R}}\right)^2$$

14. A block of mass 20 kg is placed on rough surface having co-efficient of friction 0.04 as shown in figure. Find acceleration of system when it released.




(1) 3 m/s

(2) 2 m/s

(3) 1 m/s

(4) 4 m/s

**Answer (2)**

**Sol.** Maximum friction ($F_{max}$) = 0.04 × 20 × 10 = 8N

Pulley force ($F$) = 60 N

$$\text{Acceleration } (a) = \frac{60 - 8}{26} = 2 \text{ m/s}$$

15. In single slit diffraction with slit width 0.1 mm, light of wavelength 6000 Å is used. A convex lens of focal length 20 cm is used to focus the diffracted ray. Find width of central maxima.

(1) 24 mm

(2) 2.4 mm

(3) 12 mm

(4) 1.2 mm

**Answer (2)**

**Sol.** Angular width $= \frac{2\lambda}{a}$

Linear width $= \frac{2\lambda}{a} f$

$$= \frac{2 \times 6000 \times 10^{-10} \times 20 \times 10^{-2}}{0.1 \times 10^{-3}}$$

$= 2 \times 6 \times 2 \times 10^{-4}$

$= 24 \times 10^{-4}$

$= 2.4$ mm

16.

17.

18.

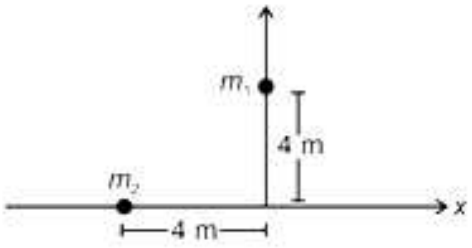
19.

20.

## SECTION - B

**Numerical Value Type Questions:** This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Two particles each of mass 2 kg are placed as shown in *xy* plane. If the distance of centre of mass from origin is $\frac{4\sqrt{2}}{x}$, find *x*



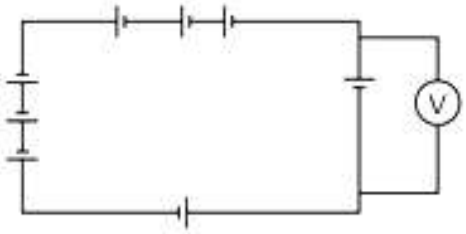

**Answer (2)**

**Sol.** $\vec{r}_{cm} = -2\hat{i} + 2\hat{j}$

$\therefore \quad r = 2\sqrt{2}$

$\boxed{x = 2}$

22. Eight identical batteries (5 V, 1 Ω) are connected as shown :




The reading of the ideal voltmeter is _____ volts.

**Answer (0)**

**Sol.** $\varepsilon = 8 \times 5 = 40$ V

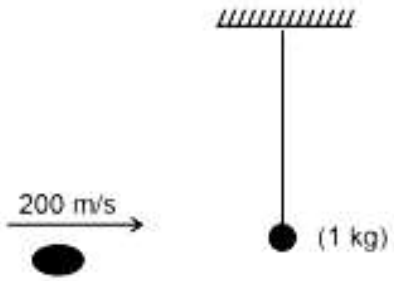
$r = 8 \times 1 = 8\ \Omega$

$\Rightarrow \quad i = 5$ A

$\Rightarrow$ Voltmeter reads

$= 5 - ir = 0$ volts

23. A bullet, of mass $10^{-2}$ kg and velocity 200 m/s gets embedded inside the bob (mass 1 kg) of a simple pendulum as shown. The maximum height the system rises by is _____ cm.




**Answer (20)**

**Sol.** Momentum conservation :

$10^{-2} \times 200 = 1 \times v \qquad ...(1)$

Energy conservation :

$v = \sqrt{2gh} \qquad ...(2)$

$\Rightarrow \quad h = \frac{v^2}{2g} = \frac{4}{20}$ m $= 20$ cm

24. The length of a seconds pendulum if it is placed at height $2R$ ($R$: radius of earth) is $\frac{10}{x\pi^2}$ metres. Find $x$.

**Answer (9)**

**Sol.** $T = 2\pi\sqrt{\frac{l}{g}}$

$\Rightarrow \quad 2 = 2\pi\sqrt{\frac{l}{g_0/9}}$

$\Rightarrow \quad 2 = 2\pi \times 3\sqrt{\frac{l}{10}}$

$\Rightarrow \quad \frac{l}{10} = \frac{1}{9\pi^2}$

$\Rightarrow \quad l = \frac{10}{9\pi^2}$ m

25. Nuclear mass and size of nucleus of an element $A$ are 64 and 4.8 femtometer. If size of nucleus of element $B$ is 4 femtometer then its nuclear mass will be $\frac{1000}{x}$ then

**Answer (27)**

**Sol.** $R^3 = \alpha A$

$\frac{(4.8^3)}{4^3} = \frac{64}{M}$

$M = \frac{16\times4\times16\times4}{48\times48\times48}\times10^3$

26. In a series LCR circuit connected to an AC source, value of the elements are $L_0$, $C_0$ & $R_0$ such that circuit is in resonance mode. If now capacity of capacitor is made $4C_0$, the new value of inductance, for circuit to still remain in resonance, is $\frac{L_0}{n}$. Find $n$.

**Answer (4)**

**Sol.** $\frac{1}{\sqrt{LC}}$ = fixed

$\Rightarrow \quad LC$ = fixed

$\Rightarrow \quad L = \frac{L_0}{4}$

27. The current through a conductor varying with time as $i = 3t^2 + 4t^3$.

Find amount of charge (in C) passes through cross section of conductor in internal $t$ = 1 sec to $t$ = 2 sec.

**Answer (22)**

**Sol.** $Q = \int i\cdot dt$

$= \int_1^2 (3t^2 + 4t^3)\cdot dt = (t^3 + t^4)_1^2$

$= (8 + 16) - (2)$

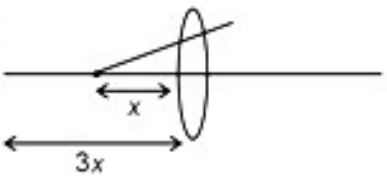
$= 22$ C

28. Distance between virtual magnified image, (size three times of object) of an object placed in front of convex lens and object is 20 cm. The focal length of lens is $x$ cm, then $x$ is _____

**Answer (15)**

**Sol.** $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ $\qquad \frac{v}{u} = 3$

$v = 3u$




$3x - x = 20$

$x = 20$

$\frac{1}{-30} - \frac{1}{-10} = \frac{1}{f}$

$\frac{1}{10} - \frac{1}{30} = \frac{1}{f}$

$\frac{2}{30} = \frac{1}{f} \Rightarrow f = 15$

29.

30.

# CHEMISTRY

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

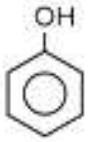
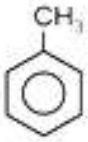
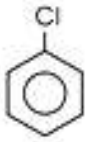
**Choose the correct answer :**

1. In Kjeldahl's estimation of nitrogen, $CuSO_4$ act as
   (1) Oxidizing agent
   (2) Reducing agent
   (3) Catalyst
   (4) Reagent

**Answer (3)**

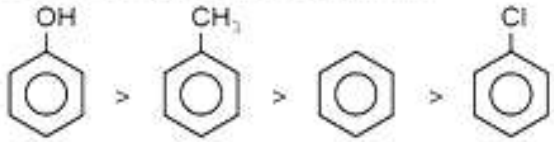
**Sol.** $CuSO_4$ acts as catalyst in Kjeldahl's method of estimation of nitrogen.

2. Which of the following is most likely attacked by electrophile?

(1) 
(2) 
(3) 
(4) 

**Answer (2)**

**Sol.** Order of reactivity towards electrophile



Strength of +M/+R : $-OH > -CH_3 > -Cl$

In case of halogens, their –I effect dominates over +M hence –Cl is deactivating and  is lesser reactive than  for incoming electrophile.

3. **Statement-I:** $PH_3$ will have low boiling point than $NH_3$.
   **Statement-II:** There are strong van der Wall forces in $NH_3$ and strong hydrogen-bonding in $PH_3$.
   (1) Statement-I and statement-II both are true
   (2) Statement-I and statement-II both are false
   (3) Statement-I is true but statement-II is false
   (4) Statement-I is false but statement-II is true

**Answer (3)**

**Sol.** Boiling point: $\overset{(239.7)}{NH_3} > \overset{(185.5)}{PH_3}$ due to hydrogen bonding in $NH_3$.

4. Which of the following have trigonal bipyramidal shape?
   $PF_5$, $PBr_5$, $[PtCl_4]^{2-}$, $SF_6$, $BF_3$, $BrF_5$, $PCl_5$, $[Fe(CO)_5]$
   (1) $PF_5$, $PBr_5$, $PCl_5$ and $Fe(CO)_5$ only
   (2) $BrF_5$, $PF_5$, $PCl_5$ and $PBr_5$ only
   (3) $PF_5$, $PCl_5$ and $[Fe(CO)_5]$ only
   (4) $[Fe(CO)_5]$, $BrF_5$, $PF_5$, $PBr_5$, $PCl_5$ only

**Answer (1)**

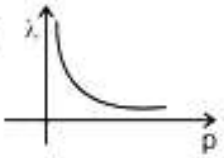
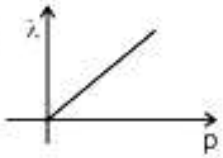
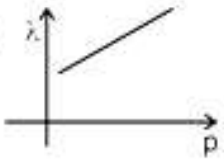
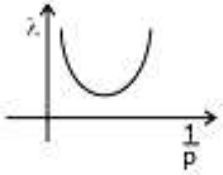
**Sol.** $PF_5$, $PCl_5$, $PBr_5$, $Fe(CO)_5 \Rightarrow$ Trigonal bipyramidal
$BrF_5 \Rightarrow$ Square pyramidal
$[PtCl_4]^{2-} \Rightarrow$ Square planar
$SF_6 \Rightarrow$ Octahedral

5. Which of the following is correct for adiabatic free expansion against vacuum
   (1) $q = 0, \Delta U = 0, W = 0$ (2) $q \neq 0, W = 0, \Delta U = 0$
   (3) $q = 0, \Delta U \neq 0, W = 0$ (4) $q = 0, \Delta U \neq 0, W \neq 0$

**Answer (1)**

**Sol.** $q = 0$ as adiabatic process is given
$W = 0$ as $p_{ext} = 0$
$q + W = \Delta U$
$q = 0$
$W = 0$
$\Rightarrow \Delta U = 0$

6. Which of the following is the correct plot between $\lambda$ (de Broglie wavelength) and p(momentum)?

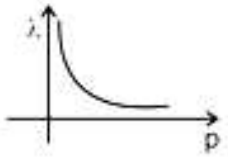
(1) 
(2) 
(3) 
(4) 

**Answer (1)**

**Sol.** $\lambda = \frac{h}{p}\left[\lambda \propto \frac{1}{p}\right]$

$\Rightarrow \lambda p = h$ (constant)

So, the plot is a rectangular hyperbola.




7. $Cr_2O_7^{2-} + xH^+ + ye^- \rightarrow 2Cr^{3+} + AH_2O$

Balance the above reaction and find x, y and A.

(1) x = 7, y = 6, A = 14 (2) x = 14, y = 6, A = 7

(3) x = 14, y = 3, A = 7 (4) x = 8, y = 2, A = 1

**Answer (2)**

**Sol.** The balanced reaction is,

$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$

x = 14

y = 6

A = 7

8. Complementary strand of DNA

ATGCTTCA is:

(1) TACGAAGA

(2) TACGAAGT

(3) TAGCAACA

(4) TAGCTACT

**Answer (2)**

**Sol.** Adenine base pairs with thymine with 2 hydrogen bonds and cytosine base pairs with guanine with 3 hydrogen bonds.

A T G C T T C A — DNA strand

T A C G A A G T → Complementary strand

9. What is the pH of $CH_3COO^-NH_4^+$ salt?

Given $K_a$ of $CH_3COOH = 1.8 \times 10^{-6}$

$K_b$ of $NH_4OH = 1.8 \times 10^{-6}$

(At 25°C)

(1) 7

(2) 9

(3) 8.9

(4) 7.8

**Answer (1)**

**Sol.** $pH = \frac{pK_w + pK_a - pK_b}{2}$

$pK_a = pK_b$

$\Rightarrow pH = \frac{pK_w}{2} = 7$

10. We are given with 3 NaCl samples and their van't Hoff factors

| Sample | van't Hoff factor |
|---|---|
| Sample-1 (0.1M) | $i_1$ |
| Sample-2 (0.01M) | $i_2$ |
| Sample-3 (0.001M) | $i_3$ |

Choose the correct answer.

(1) $i_1 = i_2 = i_3$ (2) $i_1 > i_2 > i_3$

(3) $i_3 > i_2 > i_1$ (4) $i_1 > i_3 > i_2$

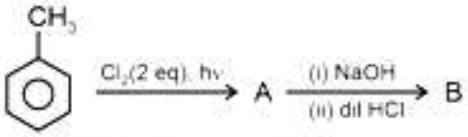
**Answer (1)**

**Sol.** As NaCl is strong electrolyte, its degree of dissociation ($\alpha$) will remain same.

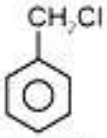
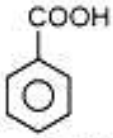
i = 2

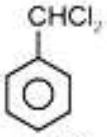
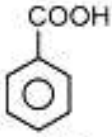
For each sample,

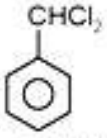
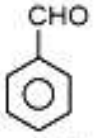
$i_1 = i_2 = i_3$

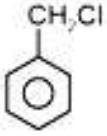
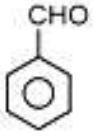
11. 


A and B in above reaction is

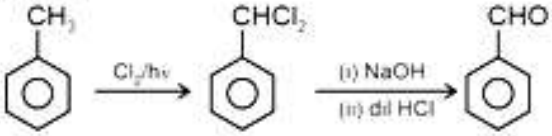
(1) (A)  (B) 

(2) (A)  (B) 

(3) (A)  (B) 

(4) (A)  (B) 

**Answer (3)**

**Sol.** 

12. We have a mixture of gases having 2 moles of monoatomic gas $\left(C_{v,m}=\frac{3R}{2}\right)$ and 6 moles of diatomic gas $\left(C_{v,m}=\frac{5R}{2}\right)$. Find out molar heat capacity ($C_{vm}$) of the mixture.

(1) $\frac{9R}{4}$

(2) $\frac{9R}{2}$

(3) 3R

(4) 4R

**Answer (1)**

**Sol.** $C_{vm}=\frac{2\left(\frac{3R}{2}\right)+6\left(\frac{5R}{2}\right)}{2+6}$

$=\frac{3R+15R}{8}=\frac{18R}{8}$

$=\frac{9R}{4}$ (option (1))

13. **Assertion (A):** KCN react with R–X to give cyanide and AgCN reacts with R – X to give isocyanide mainly.

**Reason (R):** KCN and AgCN both are ionic compounds

(1) Both Assertion and Reason are true and Reason explains Assertion

(2) Both Assertion and Reason is true but Reason does not explains Assertion

(3) Assertion is true and Reason is false

(4) Assertion is false but reason is true

**Answer (3)**

**Sol.** $KCN \longrightarrow \overset{+}{K} + CN^-$

$R-X + KCN \longrightarrow R-CN + KX$

$R-X + AgCN \longrightarrow R-NC + AgX$

KCN is ionic therefore ionised and attack occurs through carbon.

AgCN is covalent therefore attack starts with Nitrogen.

14. Consider the following two statements.

Statement I: $[Ni(H_2O)_6]^{2+}$ is of green colour

Statement II : $[Ni(CN)_4]^{2-}$ is colourless

(1) Statement I is true, statement II is false

(2) Statement I is true, statement II is true

(3) Statement I is false, statement II is true

(4) Statement I is false, statement II is false

**Answer (2)**

**Sol.** $[Ni(H_2O)_6]^{2+}$ is octahedral and $[Ni(CN)_4]^{2-}$ is square planar.

In $[Ni(H_2O)_6]^{2+} \Rightarrow Ni^{2+}$ has two unpaired electrons and in $[Ni(CN)_4]^{2-} \Rightarrow Ni^{2+}$ has no unpaired electrons.

$[Ni(H_2O)_6]^{2+}$ is coloured as it absorbs red light due to suitable d-d transition and complementary light emitted is green.

$[Ni(CN)_4]^{2-}$ has strong field ligand so the electrons of $Ni^{2+}$ pair up and it is colourless as it cannot absorb light from visible region.

15. **Statement-I:** Potassium hydrogen phthalate is primary standard for NaOH solution.

**Statement-II:** Phenolphthalein is used to detect completion of titration.

(1) Both statement-I and statement-II are correct

(2) Statement-I is correct and statement-II is incorrect

(3) Statement-I is incorrect and statement-II is correct

(4) Both statement-I and statement-II are incorrect

**Answer (1)**

**Sol.** Potassium hydrogen phthalate is used to standardize NaOH solutions.

Phenolphthalein is used as an indicator to detect completion of titrations.

16. **Statement-I:** In aniline, $-NH_2$ group is strong deactivating group for all ESR.

**Statement-II:** Aniline does not show Friedel-Craft alkylation reaction.

(1) Both statement-I and statement-II are correct

(2) Both statement-I and statement-II are incorrect

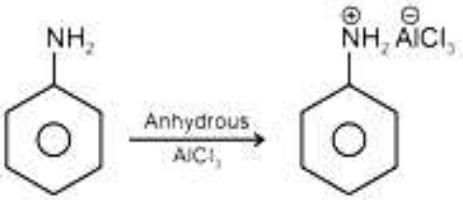
(3) Statement-I is correct and statement-II is incorrect

(4) Statement-I is incorrect and statement-II is correct

**Answer (4)**

**Sol.** In aniline $-NH_2$ is strong activating group due to presence of lone pair in nitrogen.

Aniline does not show Friedel-Craft alkylation reaction, because anhydrous $AlCl_3$ and aniline form salt together




17. Which of the following is homoleptic complex?

(1) $[Ni(CN)_4]^{2-}$

(2) $[Cu(H_2O)_3Cl_3]$

(3) $[PtCl2Br_2]^{2-}$

(4) $[Cu(NH_3)_5Cl]Cl_2$

**Answer (1)**

**Sol.** Homoleptic complexes in which a metal is bound to only one kind of donor groups/ligands.

18. For ionic reaction in organic compound which type of bond cleavage occur?

(1) Heterolytic cleavage

(2) Homolytic cleavage

(3) Free radical

(4) No cleavage of bond

**Answer (1)**

**Sol.** In heterolytic bond cleavage ions are formed. hence for ionic reaction in organic compound heterolytic bond cleavage takes place.

19. Ka values of three acids A, B and C are $10^{-3}$, $5 \times 10^{-9}$, $9 \times 10^{-11}$ respectively. The acidic strength order of these acids is

(1) A > B > C

(2) B > A > C

(3) C > B > A

(4) C > A > B

**Answer (1)**

**Sol.** Higher the value of $K_a$, more is the acidic strength.

20. Which of the following is a disproportionation reaction?

A. $Cu^{+} \longrightarrow Cu^{2+} + Cu$

B. $MnO_4^{2-} \longrightarrow MnO_4^{-} + MnO_2$

C. $H_2O_2 \longrightarrow O_2 + H_2O$

D. $CrO_4^{2-} \longrightarrow Cr^{3+} + H_2O$

(1) All A, B, C and D (2) A and B only

(3) A and C only (4) A, B and C only

**Answer (4)**

**Sol.** Disproportionation reaction is a reaction in which a substance (element) is simultaneously oxidised and reduced.

A. $\overset{\text{Reduction}}{\overset{\frown}{Cu^{+}}} \longrightarrow \overset{\text{Oxidation}}{\underset{\frown}{Cu^{2+}}} + Cu$

B. $\overset{+6}{MnO_4^{2-}} \longrightarrow \overset{+7}{MnO_4^{-}} + \overset{+4}{MnO_2}$ (Oxidation above, Reduction below)

C. $\overset{-1}{H_2O_2} \longrightarrow \overset{0}{O_2} + \overset{-2}{H_2O}$ (Reduction above, Oxidation below)

D. $\overset{+6}{CrO_4^{2-}} \longrightarrow Cr^{3+} + H_2O$ (Reduction only)

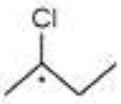
## SECTION - B

**Numerical Value Type Questions:** This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Find out total possible optical isomers of 2-chlorobutane.

**Answer (2)**

**Sol.**




There is one chiral centre present in given compound which is unsymmetrical.

Total number of isomers = $2^n$

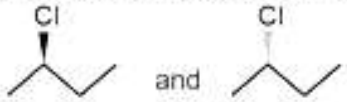
n = number of stereogenic centre

n = 1

$= 2^1$

= 2

Total two optical isomers are possible




22. We are given with following cell reaction :

$2H^+ + 2e^- \longrightarrow H_2$

$P_{H_2} = 2$ atm

$[H^+] = 1$ M

$\left(\frac{2.303RT}{F} = 0.06\right)$

If $E_{cell}$ for reaction is given by $-x \times 10^{-3}$ V, find out x.

**Answer (9)**

**Sol.** $E_{cell} = 0 - \frac{0.06}{2}\log 2$

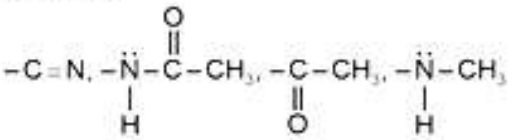
$= -0.03(0.3)$

$= -0.009$

$= -9 \times 10^{-3}$ V

x = 9

23. Total number of deactivating groups among the following



**Answer (2)**

**Sol.** $-C\equiv N$, $-\underset{\underset{H}{|}}{\ddot{N}}-\overset{\overset{O}{\|}}{C}-CH_3$, $-\overset{\overset{O}{\|}}{C}-CH_3$, $-\underset{\underset{H}{|}}{\ddot{N}}-CH_3$ are –R group which is deactivating

$-\underset{\underset{H}{|}}{\ddot{N}}-\overset{\overset{O}{\|}}{C}-CH_3$ and $-\ddot{N}H-CH_3$ due to presence of lone pair in nitrogen atom behaves as activating (+R) group.

24. How many oxides are amphoteric in nature?

$SnO_2$, $PbO_2$, $SiO_2$, $P_2O_5$, $Al_2O_3$, $CO_2$, CO, NO, $N_2O$

**Answer (3)**

**Sol.** Amphoteric oxides are those which can react with both acid and base

$SnO_2$, $PbO_2$ and $Al_2O_3$ are amphoteric oxide

$SiO_2$, $P_2O_5$, $CO_2$ are acidic oxides

CO, NO and $N_2O$ are neutral oxides

25. For carbon dating of a wood sample $\left(\frac{C^{14}}{C^{12}}\right)_t = \frac{1}{8}\left(\frac{C^{14}}{C^{12}}\right)_{t=0}$. If Half life of $C^{14}$ is 1580 years what is the life of wood sample (in yr)

**Answer (4740)**

**Sol.** $\left(\frac{C^{14}}{C^{12}}\right)_t = \frac{\left(\frac{C^{14}}{C^{12}}\right)_{t=0}}{(2)^n}$

n = 3

$t = 3 \times 1580$

= 4740 years

26. What is the minimum energy (in eV) required for an electron to excite from ground state to 1st excited state for hydrogen atom?

**Answer (10)**

**Sol.** $n_1 = 1$

$n_2 = 2$

$\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\Delta E = 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$\Delta E = 13.6\left(1 - \frac{1}{4}\right)$

$\Delta E = 13.6 \times \frac{3}{4}$ eV

= 10.05 eV ≈ 10 eV

27. Find out moles of precipitate product formed when 72 moles of $PbCl_2$ reacts with 50 moles of $(NH_4)_2SO_4$.

**Answer (50)**

**Sol.** $PbCl_2 + (NH_4)_2SO_4 \longrightarrow PbSO_4\downarrow + 2NH_4Cl$

72 50

↓ ↓ ↓

22 0 50

Moles of $PbSO_4$ formed = 50 mol

28.

29.

30.

# MATHEMATICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. If 3, $a$, $b$, $c$ are in A.P. and 3, $a-1$, $b+1$ are in G.P. Then arithmetic mean of $a$, $b$ and $c$ is

(1) 11 (2) 10

(3) 9 (4) 13

**Answer (1)**

**Sol.** 3, $a$, $b$, $c$ are in A.P.

$a - 3 = b - a$ (common diff.)

$2a = b + 3$

and 3, $a-1$, $b+1$ are in G.P.

$$\frac{a-1}{3} = \frac{b+1}{a-1}$$

$a^2 + 1 - 2a = 3b + 3$

$a^2 - 8a + 7 = 0$ $[\because\ 2a = b+3]$

$(a-7)(a-1) = 0$

If $a = 7$, $b = 2(7) - 3 = 11$, $b = 11$

and $c - b = a - 3$

$c - 11 = 4$

$c = 15$

$\therefore$ A.M of 7, 11, 15 $= \dfrac{7+11+15}{3}$

$= \dfrac{33}{3} = 11$

2. The value of $\displaystyle\int_0^{\pi/4} \frac{x\,dx}{\sin^4(2x)+\cos^4(2x)}$ is equal to

(1) $\dfrac{\pi^2}{16\sqrt{2}}$ (2) $\dfrac{\pi^2}{64}$

(3) $\dfrac{\pi^2}{32}$ (4) $\dfrac{\pi^2}{8\sqrt{2}}$

**Answer (1)**

**Sol.** $\displaystyle I = \int_0^{\pi/4} \frac{x\,dx}{\sin^4(2x)+\cos^4(2x)}$

Let $2x = t$ then $dx = \dfrac{1}{2}dt$

$$I = \int_0^{\pi/2} \frac{\frac{t}{2}\cdot\frac{1}{2}dt}{\sin^4 t + \cos^4 t}$$

$$= \frac{1}{4}\int_0^{\pi/2} \frac{t\,dt}{\sin^4 t + \cos^4 t}dt$$

$$\therefore\ I = \frac{1}{4}\int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-t\right)dt}{\sin^4 t + \cos^4 t}dt$$

$$\therefore\ 2I = \frac{1}{4}\int_0^{\pi/2} \frac{\frac{\pi}{2}dt}{\sin^4 t + \cos^4 t}$$

$$2I = \frac{\pi}{8}\int_0^{\pi/2} \frac{\sin^4 t\,dt}{\tan^4 t + 1}$$

Let $\tan t = y$ then

$$2I = \frac{\pi}{8}\int_0^{\infty} \frac{(1+y^2)dy}{1+y^4}$$

$$= \frac{\pi}{8}\int_0^{\infty} \frac{1+\frac{1}{y^2}}{y^2+\frac{1}{y^2}-2+2}dy$$

$$= \frac{\pi}{8}\int_0^{\infty} \frac{\left(1+\frac{1}{y^2}\right)dy}{2+\left(y-\frac{1}{y}\right)^2}$$

Let $y - \dfrac{1}{y} = u$

$$2I = \frac{\pi}{8}\int_{-\infty}^{\infty} \frac{du}{2+u^2}$$

$$= \frac{\pi}{8\sqrt{2}}\left[\tan^{-1}\frac{4}{\sqrt{2}}\right]_{-\infty}^{\infty}$$

$$\therefore\ I = \frac{\pi^2}{16\sqrt{2}}$$

3. If $A=\begin{bmatrix}\sqrt{2} & 1\\ -1 & \sqrt{2}\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$, $C = ABA^T$ and $X = AC^2A^T$, then $|X|$ is equal to

(1) 729 (2) 283

(3) 27 (4) 23

**Answer (1)**

**Sol.** $|A| = 3$

$|B| = 1$

$\Rightarrow |C| = |ABA^T| = |A||B|A^T| = |A|^2|B|$

$= 9$

$\Rightarrow |X| = |A||C|^2|A^T|$

$= 3 \times 9^2 \times 3 = 9 \times 9^2 = 729$

4. If 3, 7, 11, ...., 403 = $AP_1$

2, 5, 8, ...., 401 = $AP_2$

Find sum of common term of $AP_1$ and $AP_2$

(1) 3366 (2) 6699

(3) 9999 (4) 6666

**Answer (2)**

**Sol.** 3, 7, 11, 15, 19, 23, 27, ... 403 = $AP_1$

2, 5, 8, 11, 14, 17, 20, 23, ... 401 = $AP_2$

so common terms A.P.

11, 23, 35, ..., 395

$\Rightarrow 395 = 11 + (n-1)\,12$

$\Rightarrow 395 - 11 = 12\,(n-1)$

$\frac{384}{12} = n-1$

$32 = n - 1$

$n = 33$

Sum $= \frac{33}{2}[2\times11+(32)12]$

$= \frac{33}{2}[22+384]$

$= 6699$

5. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{\left(1+e^{\sin x}\right)\left(1+\sin^4 x\right)}dx = a\pi + b\log\left(3+2\sqrt{2}\right)$

then find $a + b$.

(1) 4 (2) 6

(3) 8 (4) 2

**Answer (1)**

**Sol.** $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{\left(1+e^{\sin x}\right)\left(1+\sin^4 x\right)}dx$

$= \int_0^{\frac{\pi}{2}} \left\{\frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)} + \frac{8\sqrt{2}\cos x}{(1+e^{-\sin x})(1+\sin^4 x)}\right\}dx$

$= 8\sqrt{2}\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^4 x}dx$

Let $\sin x = t$

$I = 8\sqrt{2}\int_0^1 \frac{dt}{1+t^4}$

$= 4\sqrt{2}\int_0^1 \frac{\left(1+\frac{1}{t^2}\right)-\left(1-\frac{1}{t^2}\right)}{t^2+\frac{1}{t^2}}dt$

$= 4\sqrt{2}\int_0^1 \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+2} - 4\sqrt{2}\int_0^1 \frac{\left(1-\frac{1}{t^2}\right)dt}{\left(t+\frac{1}{t}\right)^2-2}$

$= 4\sqrt{2}\cdot\frac{1}{\sqrt{2}}\left(\tan^{-1}\frac{t-\frac{1}{t}}{\sqrt{2}}\right)_0^1 - 4\sqrt{2}\cdot\frac{1}{2\sqrt{2}}\left[\log\left|\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right|\right]_0^1$

$= 2\pi - 2\log\left|\frac{2-\sqrt{2}}{2+\sqrt{2}}\right|$

$= 2\pi + 2\log\left(3+2\sqrt{2}\right)$

$\therefore \quad a = b = 2$

6. If $(t + 1)dx = (2x + (t + 1)^3)dt$ and $x(0) = 2$, then $x(1)$ is equal to

(1) 5 (2) 12

(3) 6 (4) 8

**Answer (2)**

**Sol.** $(t + 1)dx = (2x + (t + 1)^3)dt$

$\therefore \frac{dx}{dt} - \frac{2x}{t+1} = (t+1)^2$

$\therefore$ I.F. $= e^{\int -\frac{2}{t+1}dt} = \frac{1}{(t+1)^2}$

$\therefore$ Solution is

$\frac{x}{(t+1)^2} = \int 1dt$

$x = (t + c)\,(t + 1)^2$

$\because x(0) = 2$ then $c = 2$

$\therefore x = (t+2)(t+1)^2$

$\therefore x(1) = 12$

7. Five people are distributed in four identical rooms. A room can also contain zero people. Find the number of ways to distribute them.

(1) 47 (2) 53

(3) 43 (4) 51

**Answer (4)**

**Sol.** Total ways to partition 5 into 4 parts are:

$5\,0\,0\,0 \rightarrow 1$

$4\,1\,0\,0 \rightarrow \frac{5!}{4!} = 5$

$3\,2\,0\,0 \rightarrow \frac{5!}{3!\cdot 2!} = 10$

$3\,1\,1\,0 \rightarrow \frac{5!}{3!\cdot 2!} = 10$

$2\,2\,1\,0 \rightarrow \frac{5!}{2!2!2!} = 15$

$2\,1\,1\,1 \rightarrow \frac{5!}{2!\times 3!} = 10$

$51 \rightarrow$ Total way

8. $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 4$ and $y = 9f(x)\cdot x^2$. If $y$ is strictly increasing function, find interval of $x$.

(1) $\left(-\infty, \frac{-1}{\sqrt{5}}\right] \cup \left(\frac{-1}{\sqrt{5}}, 0\right)$

(2) $\left(\frac{-1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

(3) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(4) $\left(-\sqrt{\frac{2}{5}}, 0\right) \cup \left(\sqrt{\frac{2}{5}}, \infty\right)$

**Answer (4)**

**Sol.** $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 4$ ...(1)

Replace $x$ by $\frac{1}{x}$

$5f\left(\frac{1}{x}\right) + 4f(x) = \frac{1}{x^2} - 4$ ...(2)

5 × equation (1) – 4 × equation (2)

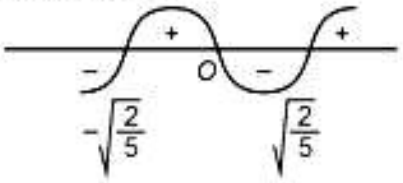
$9f(x) = 5x^2 - \frac{4}{x^2} - 4$

$y = 9f(x)\cdot x^2 = \frac{5x^4 - 4 - 4x^2}{x^2}x^2$

$y = 5x^4 - 4 - 4x^2$

$y' = 20x^3 - 8x > 0$

$4x(5x^2 - 2) > 0$




$x \in \left(-\sqrt{\frac{2}{5}}, 0\right) \cup \left(\sqrt{\frac{2}{5}}, \infty\right)$

9. If hyperbola $x^2 - y^2\text{cosec}^2\theta = 5$ and ellipse $x^2\text{cosec}^2\theta + y^2 = 5$ has eccentricity $e_H$ and $e_E$ respectively and $e_H = \sqrt{7}e_E$, then $\theta$ is equal to

(1) $\frac{\pi}{3}$

(2) $\frac{\pi}{6}$

(3) $\frac{\pi}{2}$

(4) $\frac{\pi}{4}$

**Answer (1)**

**Sol.** $x^2 - y^2\text{cosec}^2\theta = 5 \quad \Rightarrow \frac{x^2}{1} - \frac{y^2}{\sin^2\theta} = 5$

$x^2\text{cosec}^2\theta + y^2 = 5 \quad \Rightarrow \frac{x^2}{\sin^2\theta} + \frac{y^2}{1} = 5$

$e_H = \sqrt{7}e_E$

$e_H = \sqrt{1 + \frac{\sin^2\theta}{1}}$

and $e_E = \sqrt{1 - \frac{\sin^2\theta}{1}}$

$\Rightarrow \sqrt{1 + \sin^2\theta} = \sqrt{7}\sqrt{1 - \sin^2\theta}$

$\Rightarrow 1 + \sin^2\theta = 7 - 7\sin^2\theta$

$\Rightarrow 8\sin^2\theta = 6$

$\Rightarrow \sin\theta = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$

10. A bag contains 8 balls (black and white). If four balls are chosen without replacement then $2W$ and $2B$ are found then the probability that number of white and black balls are same in bag is equal to

(1) $\frac{1}{7}$ (2) $\frac{2}{7}$

(3) $\frac{3}{5}$ (4) $\frac{1}{2}$

**Answer (2)**

**Sol.** $P(2W \text{ and } 2B) = P(2B, 6W) \times P(2W \text{ and } 2B)$
$+ P(3B, 5W) \times P(2W \text{ and } 2B)$
$+ P(4B, 4W) \times P(2W \text{ and } 2B)$
$+ P(5B, 3W) \times P(2W \text{ and } 2B)$
$+ P(6B, 2W) \times P(2W \text{ and } 2B)$

$$= \frac{1}{9}\left\{0+0+\frac{{}^2C_2 \times {}^6C_2}{{}^8C_4}+\frac{{}^3C_2 \cdot {}^5C_2}{{}^8C_4}+\frac{{}^4C_2 \cdot {}^4C_2}{{}^8C_2} + \frac{{}^5C_2 \cdot {}^3C_2}{{}^8C_4}+\frac{{}^6C_2 \cdot {}^2C_2}{{}^8C_4}+0+0\right\}$$

$$= \frac{1}{9} \times \frac{1}{{}^8C_4}(15+30+36+30+15)$$

$$= \frac{1}{9} \times \frac{1}{{}^8C_4} \times 126$$

$$P\left(\frac{4B \text{ and } 4W}{2W \text{ and } 2B}\right) = \frac{\frac{1}{9} \times \frac{{}^4C_2 \times {}^4C_2}{{}^8C_4}}{\frac{1}{9} \times \frac{1}{{}^8C_4} \times 126}$$

$$= \frac{36}{126}$$

$$= \frac{18}{63}$$

$$= \frac{6}{21}$$

$$= \frac{2}{7}$$

11. If two circle $x^2 + y^2 = 4$ and $x^2 + y^2 - 4\lambda x + 9 = 0$ intersect at two distinct points, then find the range of $\lambda$.

(1) $\left(-\infty, -\frac{13}{2}\right) \cup \left(\frac{13}{2}, \infty\right)$

(2) $\left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$

(3) $\left[-\frac{13}{8}, \frac{13}{8}\right]$

(4) $\lambda \in \left(\frac{3}{2}, \infty\right)$

**Answer (2)**

**Sol.** $|r_1 - r_2| < c_1c_2 < r_1 + r_2$

$$\Rightarrow \left|2 - \sqrt{4\lambda^2 - 9}\right| < |2\lambda| < 2 + \sqrt{4\lambda^2 - 9}$$

$$\Rightarrow |2\lambda| - 2 < \sqrt{4\lambda^2 - 9}$$

$$\Rightarrow 4\lambda^2 + 4 - 8|\lambda| < 4\lambda^2 - 9$$

$$\lambda > \frac{13}{8}, \lambda < -\frac{13}{8}$$

$$\sqrt{4\lambda^2 - 9} > 0$$

$$\Rightarrow \lambda > \frac{3}{2}, \lambda < -\frac{3}{2}$$

$$\therefore \lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$$

Now,

$$\left|2 - \sqrt{4\lambda^2 - 9}\right| < |2\lambda|$$

$$\Rightarrow 4 + 4\lambda^2 - 9 - 4\sqrt{4\lambda^2 - 9} < 4\lambda^2$$

$$\Rightarrow 4\sqrt{4\lambda^2 - 9} > -5 \Rightarrow \lambda \in R$$

$$\therefore \lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$$

12. If $S = \left\{x \in R : 3\left(\sqrt{3}+\sqrt{2}\right)^x + \left(\sqrt{3}-\sqrt{2}\right)^x = \frac{10}{3}\right\}$ then number of elements in set $S$ is

(1) Zero (2) 1

(3) 2 (4) 3

**Answer (3)**

**Sol.** $\sqrt{3} - \sqrt{2} = \frac{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)} = \frac{1}{\sqrt{3}+\sqrt{2}}$

Let $\sqrt{3} + \sqrt{2} = t$

$$\Rightarrow t^x + \frac{1}{t^x} = \frac{10}{3}$$

Let $t^x = y \Rightarrow y + \frac{1}{y} = \frac{10}{3}$

$$\Rightarrow y = 3 \text{ or } \frac{1}{3}$$

$$\Rightarrow \left(\sqrt{3}+\sqrt{2}\right)^x = 3 \text{ or } \frac{1}{3}$$

$$x\log\left(\sqrt{3}+\sqrt{2}\right) = \ln 3 \text{ or } -\ln 3$$

$$\Rightarrow x = \frac{\ln 3}{\ln\left(\sqrt{3}+\sqrt{3}\right)} \text{ or } \frac{-\ln 3}{\sqrt{3}+\sqrt{2}}$$

$\Rightarrow$ two real values of $x$

13. $f(x)=\begin{cases}e^{-x} & , x<0\\ \ln x & , x>0\end{cases}$

$g(x)=\begin{cases}e^{x} & , x<0\\ x & , x>0\end{cases}$

The $gof: A \to R$ is

(1) Onto but not one-one

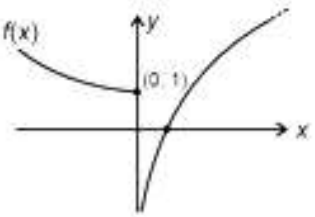
(2) Into and many one

(3) Onto and one-one

(4) Into and one-one

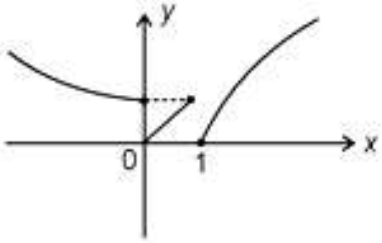
**Answer (2)**

**Sol.**




$gof(x)=\begin{cases}f(x) & , f(x)<0\\ f(x) & , f(x)>0\end{cases}$

$=\begin{cases}e^{\ln x}=x & (0,1)\\ e^{-x} & (-\infty,0)\\ \ln x & (1,\infty)\end{cases}$




$\therefore$ $gof(x)$ is many one and into

14. If $\tan A=\dfrac{1}{\sqrt{x^2+x+1}}$, $\tan B=\dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C=\dfrac{1}{\sqrt{x(x^2+x+1)}}$, then $A+B=$

(1) 0 (2) $\pi-C$

(3) $\dfrac{\pi}{2}-C$ (4) None

**Answer (3)**

**Sol.** $\tan B\times\tan C=\dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}\times\dfrac{1}{\sqrt{x(x^2+x+1)}}$

$=\dfrac{1}{x^2+x+1}=\tan^2 A$

$\tan^2 A=\tan B\tan C$

It is only possible when $A=B=C$ at $x=1$

$\Rightarrow$ $A=30^\circ, B=30^\circ, C=30^\circ$

$\left[\tan A=\tan B=\tan C=\dfrac{1}{\sqrt{3}}\right]$

$\therefore$ $A+B=\dfrac{\pi}{2}-C$

15. $\lim\limits_{x\to 0}\dfrac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}$, where $\{\}$ is fractional part function.

If L.H.L = $L$ and R.H.L = $R$, then the correct relation between $L$ and $R$ is

(1) $\sqrt{2}R=4L$ (2) $\sqrt{2}L=4R$

(3) $R=L$ (4) $R=2L$

**Answer (1)**

**Sol.** $RHL\Rightarrow\lim\limits_{x\to 0^+}\dfrac{\cos^{-1}(1-x^2)\sin^{-1}(1-x)}{x-x^3}$

$\Rightarrow\lim\limits_{x\to 0^+}\dfrac{\pi}{2}\cdot\dfrac{\cos^{-1}(1-x^2)}{x}$

$\dfrac{\pi}{2}\lim\limits_{x\to 0^+}\dfrac{-1}{\sqrt{1-(1-x^2)^2}}(-2x)$

$=\dfrac{\pi}{2}\lim\limits_{x\to 0^+}\dfrac{2x}{\sqrt{2x^2-x^4}}=\pi\lim\limits_{x\to 0^+}\dfrac{x}{x\sqrt{2-x^2}}$

$=\dfrac{\pi}{\sqrt{2}}$

$LHL\Rightarrow\lim\limits_{x\to 0^-}\dfrac{\cos^{-1}\left(1-(1+x)^2\right)\sin^{-1}(1-(1+x))}{1\cdot\left(1-(1+x)^2\right)}$

$=\lim\limits_{x\to 0^-}\dfrac{\cos^{-1}(-x^2-2x).\sin^{-1}(-x)}{-x^2-2x}$

$=\dfrac{\pi}{2}\lim\limits_{x\to 0^-}\dfrac{-\sin^{-1}x}{-x(x+2)}=\dfrac{\pi}{2}\times\dfrac{1}{2}=\dfrac{\pi}{4}$

16.

17.

18.

19.

20.

## SECTION - B

**Numerical Value Type Questions:** This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Let $S=\{1, 2, 3, \ldots, 20\}$

$R_1=\{(a, b): a \text{ divide } b\}$

$R_2=\{(a, b): a \text{ is integral multiple of } b\}$ $a, b \in s$

$n(R_1-R_2)=?$

**Answer (46)**

**Sol.** $R_1$ = {(1, 1), (1, 2), (1, 3)...., (1, 20), (2, 2), (2, 4)....(2, 20), (3, 3), (3, 6), ....(3, 18), (4, 4), (4, 8), ... (4, 20), (5, 5), (5, 10), (5, 15), (5, 20), (6, 6), (6, 12), (6, 18), (7, 7), (7, 14), (8, 8), (8, 16), (9, 9), (9, 18), (10, 10), (10, 20), (11, 11), (12, 12), ....(20, 20)}

$n(R_1) = 66$

$R_2$ = {$a$ is integral multiple of $b$}

So $n(R_1 - R_2) = 66 - 20 = 46$

as $R_1 \cap R_2 = \{(a, a) : a \in s\} = \{(1, 1), (2, 2)..., (20, 20)\}$

22. The number of solution of equation $x + 2y + 3z = 42$ and $x, y, z \in z$ and $x, y, z \geq 0$ is

**Answer (168)**

**Sol.** $x + 2y + 3z = 42$

0 $x + 2y = 42 \Rightarrow$ 22 cases

1 $x + 2y = 39 \Rightarrow$ 19 cases

2 $x + 2y = 36 \Rightarrow$ 19 cases

3 $x + 2y = 33 \Rightarrow$ 17 cases

4 $x + 2y = 30 \Rightarrow$ 16 cases

5 $x + 2y = 27 \Rightarrow$ 14 cases

6 $x + 2y = 24 \Rightarrow$ 13 cases

7 $x + 2y = 21 \Rightarrow$ 11 cases

8 $x + 2y = 18 \Rightarrow$ 10 cases

9 $x + 2y = 15 \Rightarrow$ 8 cases

10 $x + 2y = 12 \Rightarrow$ 7 cases

11 $x + 2y = 9 \Rightarrow$ 5 cases

12 $x + 2y = 6 \Rightarrow$ 4 cases

13 $x + 2y = 3 \Rightarrow$ 2 cases

14 $x + 2y = 0 \Rightarrow$ 1 cases

23.

24.

25.

26.

27.

28.

29.

30.

❑ ❑ ❑